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Quenching Behavior for a Nonlinear Parabolic Equation with Nonlinear Boundary Flux

Zhe Jia[1] and Zuodong Yang[1,2*]

[1] *Institute of Mathematics, School of Mathematics Science, Nanjing Normal University, Jiangsu Nanjing 210023, China.*

[2] *School of Teacher Education, Nanjing Normal University, Jiangsu Nanjing 210097, China.*

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

The paper deals with a nonlinear equation in one-dimensional space, of which the nonlinearity appears both in source term and the Neumann boundary condition. Firstly, we proved that the solution of problem (1.1) quenches in finite time and the only quenching point is $x = 0$ if the initial data is appropriate. Then we established the corresponding quenching rate of the solution.

Keywords: Nonlinear parabolic equation; quenching time; quenching rate; nonlinear boundary flux.

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**Corresponding author: E-mail: zdyang_jin@263.net;*

1 Introduction

In this paper, we are concerned with the blow-up phenomenon of the following problem:

$$\begin{array}{ll} u_t = f(u)(\mid u_x \mid^{p-2} u_x)_x + (1-u)^{-h}, & 0 < x < 1, t > 0, \\ u_x(0,t) = u^{-q}(0,t),\ u_x(1,t) = 0, & t > 0, \\ u(x,0) = u_0(x), & 0 \le x \le 1, \end{array} \tag{1.1}$$

where $f(u)$ is a monotone decreasing function with $f(u) > 0$ for $u > 0$. $p \ge 2$, h, q are positive constants. In addition, $0 < u_0(x) < 1$ for $x \in (0,t)$ and u_0 satisfies the compatibility conditions.

At first, we give the definition about quenching: we claim that the solution of the problem (1.1) quenches in finite time, which there exists a $0 < T < \infty$, such that

$$\lim_{t\to T^-} \min_{0\le x\le 1} u(x,t) = 0 \ \text{ or } \ \lim_{t\to T^-} \max_{0\le x\le 1} u(x,t) = 1.$$

Kawarada first studied the quenching behavior of the semilinear heat equation $u_t = u_{xx} + 1/(1-u)$ in 1975(see[1]). Since then, there are many conclusions on the quenching phenomenon(see[2]-[5],[6]-[12]). Quenching phenomenon depends on the singular term of the problem. For example, Zhi and Mu in [13] considered a problem with nonlinear boundary outflux at one side:

$$\begin{array}{ll} u_t = u_{xx} + (1-u)^{-p}, & 0 < x < 1, \ \ t > 0, \\ u_x(0,t) = u^{-q}(0,t), \ \ u_x(1,t) = 0, & t > 0 \\ u(x,0) = u_0(x), & 0 \le x \le 1. \end{array} \tag{1.2}$$

They obtained that u quenches in finite time T, and the only quenching point is $x = 0$, and they also show the quenching rate near the quenching time T. In [14], Selcuk.B and Ozalp.N discussed the same equation from (1.2), but the Neumann boundary condition is $u_x(0,t) = 0, u_x(1,t) = -u^{-q}(1,t), \ \ t > 0$. They showed that the only quenching point is $x = 1$ and gave the quenching rate.

In addition, K.Deng and M.Xu [15] considered the following problem:

$$\begin{array}{ll} (\psi(u))_t = u_{xx}, & 0 < x < 1, \ \ t > 0, \\ u_x(0,t) = 0, \ \ u_x(1,t) = u^{-q}(0,t), & t > 0 \\ u(x,0) = u_0(x) & 0 \le x \le 1 \end{array} \tag{1.3}$$

They obtained that the finite time quenching for the solution and established results about quenching set and rate.

In recent year, there are more and more people researched the quenching phenomenon for degenerate parabolic problem see([16]-[21]).

In [16], Yang.Y etc study the following problem:

$$\begin{array}{ll} u_t = (|u_x|^{p-2}|u_x|)_x, & 0 < x < 1, \ \ t > 0, \\ u_x(0,t) = 0, \ \ u_x(1,t) = -g(u(1,t))), & t > 0, \\ u(x,0) = u_0(x), & 0 \le x \le 1, \end{array} \tag{1.4}$$

They showed that the quenching occurs only at $x = 1$ and gave the bounds for the quenching rate.

Recently, Ying Yang [22] researched a non-Newtonian filtration equation with singular boundary flux:

$$\begin{array}{ll} u_t = (|u_x|^{p-2}|u_x|)_x + (1-u)^{-h}, & 0 < x < 1, \ \ t > 0, \\ u_x(0,t) = 0, \ \ u_x(1,t) = -u^{-q}(1,t), & t > 0, \\ u(x,0) = u_0(x), & 0 \le x \le 1, \end{array} \tag{1.5}$$

They got the solution quenched in a finite time and the time derivative blow up at the quenching point. They also give the quenching rate.

Motivated by the results of the above cited papers. By using methods in [15, 16, 22], the results of the literature [22]are extended to the problem (1.1). It is easy to see that there are two singularity terms. The specific structure is as follows. At first, in section 2 we show that the finite-time quenching occurs for appropriate initial data and the only quenching point is $x = 0$. Then in section 3 we calculate the quenching rate.

Throughout this paper, we assume that:

(H_1) $u_0'(x) \ge 0$,

(H_2) $f(u(0,x))(| u_0'(x) |^{p-2} u_0'(x))_x + (1-u(0,x))^{-h} < 0.$

2 Quenching on the Boundary

In this section, we will prove finite time quenching for the solution. In virtue of the degeneracy of the equation, the classical solutions might not exist in general, so we should discuss weak solutions. However for simplifying our arguments, we suppose that the solution is appropriately smooth, since we may consider some approximate boundary and initial value conditions.

Lemma 2.1. Assume that $(H_1), (H_2)$ hold and u is the solution of problem (1.1) in $(0, T_0)$, and $T_0 > 0$. Then $u_x(x,t) \ge 0$ and $u_t(x,t) < 0$ in $(0,1) \times (0, T_0)$.

Proof. Denote $\omega = u_t$. Then $\omega(x,t)$ satisfies

$$\begin{array}{ll} \omega_t = (p-1)f(u)(| u_x |^{p-2} \omega_x)_x + f'(u)(| u_x |^{p-2} u_x)_x \omega + h(1-u)^{-h-1}\omega, & 0 < x < 1, 0 < t < T_0, \\ \omega_x(0,t) = -qu^{-q-1}(0,t)\omega(0,t), \quad \omega_x(1,t) = 0, & 0 < t < T_0, \\ \omega(x,0) = u_t(x,0) = f(u(0,x))(| u_0'(x) |^{p-2} u_0'(x))_x + (1-u)^{-h} < 0, & 0 \le x \le 1, \end{array}$$

The maximum principle leads to $\omega < 0$, and thus $u_t < 0$ in $(0,1] \times (0, T_0)$ for $T_0 > 0$.

Similarity, letting $\nu = u_x$, we have

$$\begin{array}{ll} \nu_t = f(u)(| \nu |^{p-2} \nu)_{xx} + f'(u)\nu(| \nu |^{p-2} \nu)_x + h(1-u)^{-h-1}\nu, & 0 < x < 1, 0 < t < T_0 \\ \nu(0,t) = u^{-q}(0,t), \quad \nu(1,t) = 0, & 0 < t < T_0, \\ \nu(x,0) = u_0'(x), & 0 \le x \le 1, \end{array} \tag{2.1}$$

The maximum principle leads to $u_x \ge 0$ in $(0,1] \times (0, T_0)$. Then it is easy to conclude that the problem (2.1) is not degenerate in $(0,1] \times (0, T_0)$. So u_x is a classical solution of (2.1). Therefore, the solution of the problem (1.1) $u \in C^{2,1}((0,1] \times (0, T_0))$,and $u_x(x,t) \ge 0, u_t(x,t) < 0$ in $(0,1] \times (0, T_0)$. The proof of lemma 2.1 is complete. □

Theorem 2.1. Assume that $(H_1), (H_2)$ hold, then every solution of (1.1) quenching in finite time, and the only quenching point is $x = 0$.

Proof. By lemma 2.1, we have

$$\min_{0 \le x \le 1} u(x,t) = u(0,t), \quad \min_{0 \le x \le 1} v(x,t) = v(0,t).$$

Since $u_t < 0$,there exists a positive constant γ such that $u_t \le -\gamma$. Define $L(t) = \int_0^1 u(x,t)dx$, so it is easy to see that

$$L'(t) = \int_0^1 u_t dx \le -\gamma.$$

Thus $L(t) \le L(0) - \gamma t$, which means there exists T such that $u(0,t) \to 0$ for $t \to T^-$. Next, we will prove u must quench at $x = 0$. In what follows, we only need to prove the quenching can not occur in $(0, 1/4) \times (\eta, T)$ for $\eta(0 < \eta < T)$. Denote

$$H(x,t) = u_x - \varepsilon(1/4 - x), \quad (x,t) \in (0,1/4) \times (\eta, T),$$

where ε is a positive constant to be specified later. Since $f(u) > 0, f'(u) < 0, u_x(x,t) > 0$ in $(0,1) \times (0,T)$. So $H(x,t)$ satisfies

$$\begin{aligned} H_t &= u_{xt} \\ &= f(u)(| u_x |^{p-2} u_x)_{xx} + f'(u)u_x(| u_x |^{p-2} u_x)_x + h(1-u)^{-h-1}u_x \\ &= (p-1)(p-2)f(u)u_x^{p-3}u_{xx}^2 + (p-1)f(u)u_x^{p-2}u_{xxx} \\ &\quad +(p-1)f'(u)u_x^{p-1}u_{xx} + h(1-u)^{-h-1}u_x \\ &= (p-1)f(u)u_x^{p-2}H_{xx} + (p-1)(p-2)f(u)u_x^{p-3}(H_x - \varepsilon)^2 \\ &\quad +(p-1)f'(u)u_x^{p-3}(H_x - \varepsilon) + h(1-u)^{-h-1}u_x \\ &= (p-1)f(u)u_x^{p-2}H_{xx} + (p-1)f'(u)u_x^{p-3}H_x + (p-1)(p-2)f(u)u_x^{p-3}(H_x \\ &\quad -\varepsilon)^2 - (p-1)f'(u)u_x^{p-3}\varepsilon + h(1-u)^{-h-1}u_x, \end{aligned}$$

for $(x,t) \in (0,1/4) \times (\eta, T)$.

It means

$$H_t - (p-1)f(u)u_x^{p-2}H_{xx} - (p-1)f'(u)u_x^{p-3}H_x \ge 0.$$

On the parabolic boundary,

$$\begin{array}{ll} H(0,t) > 0, \ \ H(1/4,t) > 0, & t \in (\eta, T), \\ H(x,\eta) > 0, & x > 0, \end{array}$$

provided ε is sufficiently small. By the maximum principle, we find that

$$H(x,t) \le 0, \quad (x,t) \in (0,1/4) \times (\eta, T),$$

So $u(x,t) > 0$, if $x > 0$. The proof of Theorem 2.1 is complete. □

Remark 2.1. From Theorem 1.1, we can see that the case of

$$\lim_{t \to T^-} \max_{0 \le x \le 1} u(x,t) = 1,$$

will not occur as a result of our choice of the initial datum.

3 Bounds for Quenching Rate

In this section, we establish bounds on the quenching rate. We first present the upper bound.

Theorem 3.1. Assume that the hypothesis of Theorem (2.1) hold, then there exists a positive constant C_1 such that

$$\int_0^{u(0,t)} \frac{s^{qp+1}}{f(s)} ds \leq C_1 q(T-t).$$

for t sufficiently close to T.

Proof. we define a function $E(x,t) = |u_x(x,t)|^{p-2}u_x(x,t) - \varphi^{p-1}(x,t)u^{-q(p-1)}(x,t)$ in $(0,1)\times(0,T)$. Here $\varphi(x)$ is given as follows:

$$\varphi(x) = \begin{cases} \frac{(x_0-x)^r}{x_0^r}, & x \in [0,x_0], \\ 0, & x \in (x_0,1], \end{cases}$$

with some $x_0 < 1$ and choosing $r > 3$ large enough so that $\varphi(x) \leq u_0^{'}(x)u_0^q(x)$ for $0 \leq x \leq x_0$. It is easy to see $E(0,t) = E(1,t) = 0$ and $E(x,0) \geq 0$.

In addition, $E(x,t)$ also satisfies

$$\begin{aligned} E_t &= (p-1)u_x^{p-2}f(u)E_{xx} + (p-1)u_x^{p-1}f^{'}(u)E_x \\ &+(p-1)^2 f(u)u_x^{p-2}u^{-q(p-1)}[(p-2)\varphi^{p-3}(x)(\varphi^{'})^2(x) + \varphi^{p-2}(x)\varphi^{''}(x)] \\ &+q(p-1)(1-u)^{-h}\varphi^{p-1}(x)u^{-q(p-1)-1} + q(p-1)^2[q(p-1)+1]u_x^p f(u)\varphi^{p-1}(x)u^{-q(p-1)-2} \\ &+(p-1)^2u_x^{p-1}f^{'}(u)\varphi^{p-2}(x)\varphi^{'}(x)u^{-q(p-1)} - q(p-1)^2u_x^p\varphi^{p-1}(x)u^{-q(p-1)-1} \\ &-2q(q-1)^3u^{p-1}(x)f(u)\varphi^{p-2}(x)\varphi^{'}(x)u^{-q(p-1)-1} \\ &+(p-1)u_x^{p-1}h(1-u)^{-h-1}. \end{aligned}$$

According to the definition of $\varphi(x)$, it is easy to see that $\varphi(x) \geq 0, \varphi^{'}(x) \leq 0, \varphi^{''}(x) \geq 0$. Then we have

$$E_t - (p-1)u_x^{p-2}f(u)E_{xx} - (p-1)u_x^{p-1}f^{'}(u)E_x \geq 0 \qquad (3.1).$$

Thus, by the maximum principle, we can see that $E(x,t) \geq 0$, that is

$$u_x(x,t) \geq \varphi(x)u^{-q}, \quad (x,t) \in [0,1]\times[0,T). \qquad (3.2)$$

It is easy to see that

$$E_x(0,t) = \lim_{x\to 0^+} \frac{E(x,t)-E(0,t)}{x} \geq 0, \qquad (3.3)$$

which means

$$\begin{aligned} u_t(0,t) &\geq (p-1)f(u(0,t))\varphi^{'}(0)u^{-q(p-1)}(0,t) - q(p-1)f(u(0,t))u^{-qp-1}(0,t) + (1-u)^{-h} \\ &\geq (\tfrac{\varphi^{'}(0)}{q} - 1)[q(p-1)f(u(0,t))u^{-qp-1}(0,t)] \\ &\geq -\widetilde{c}q(p-1)f(u(0,t))u^{-qp-1}(0,t), \end{aligned} \tag{3.4}$$

where $\widetilde{c}(\geq 1 - \frac{\varphi^{'}(0)}{q})$ is a positive constant.

Integrating (3.4) from t to T, we get

$$\int_0^{u(0,t)} \frac{s^{qp+1}}{f(s)} ds \leq C_1(T-t),$$

where $C_1 = \widetilde{c}q(p-1)$.

The proof of Theorem 3.1 is complete. □

Next we will give the lower bound on the quenching rate, the derivation of which is in the spirit of [15]. We need the following additional hypotheses: there exist a constant δ $(-\infty < \delta \leq 2 - 1/(p-1))$such that

$$(H_3)\ (-qf(u)u^{-q(p-1)(\delta-1)-q-1})^{''} < 0.$$

Theorem 3.2. Assume that the hypotheses $(H_1), (H_2), (H_3)$ hold, then there exists a positive constant C_2 such that

$$\int_0^{u(0,t)} \frac{s^{qp+1}}{f(s)} ds \geq C_2 q(T-t).$$

Proof. Let$k(u) = -qf(u)u^{-q(p-1)(\delta-1)-q-1}$. Notice that the hypothesis (H_3) implies that

$$(\widetilde{H_3})\ \ k^{''}(u) \leq 0.$$

Set $\psi(x,t) = u_t - \varepsilon k(u)u_x^{(p-1)(2-\delta)}$ in $(0, T-\tau) \times (\tau, T)$, where ε is a positive constant. After some calculation we have

$$\psi_t = (p-1)f(u)u_x^{p-2}\psi_{xx} + (p-1)(p-2)f(u)u_x^{p-3}u_{xx}\psi_x + J(x,t)\psi + Q(x,t),$$

where

$$\begin{aligned} J(x,t) = &[\tfrac{f^{'}(u)}{f(u)} + \varepsilon(2-\delta)(2p-\delta p+\delta-3)\tfrac{k(u)}{f(u)}u_x^{p-\delta p+\delta-2}]u_t, \\ &-[\varepsilon(2-\delta)(2p-\delta p+\delta-3)\tfrac{p}{p-1}\tfrac{k(u)}{f(u)}u_x^{p-\delta p+\delta-2} + \tfrac{f^{'}(u)}{f(u)}] \\ &+\varepsilon[(5p-2\delta p+2\delta-6)k^{'}(u) - (2p-\delta p+\delta-3)\tfrac{k(u)}{f(u)}f^{'}(u)]u_x^{(p-1)(2-\delta)} \\ &+\varepsilon^2(2-\delta)(2p-\delta p+\delta-3)\tfrac{k^2(u)}{f(u)}u_x^{3p-2\delta p+2\delta-4} \\ &+h(1-u)^{-h-1}, \end{aligned} \tag{3.5}$$

$$
\begin{aligned}
Q(x,t) = {} & \varepsilon^2(2-\delta)(2p-\delta p+\delta-3)\tfrac{k^3(u)}{f(u)}u_x^{5p-3\delta p+3\delta-6} \\
& +\varepsilon^2 k(u)[(5p-2\delta p+2\delta-6)k'(u)-\tfrac{k(u)}{f(u)}f'(u)(2p-\delta p+\delta-3)]u_x^{2(p-1)(2-\delta)} \\
& +\tfrac{1}{p-1}\varepsilon(2-\delta)(2p-\delta p+\delta-3)\tfrac{k(u)}{f(u)}u_x^{p-\delta p+\delta-2}(1-u)^{-2h} \\
& -\varepsilon^2(2-\delta)(2p-\delta p+\delta-3)\tfrac{k^2(u)}{f(u)}u_x^{3p-2\delta p+2\delta-4}(1-u)^{-h} \\
& +\varepsilon(p-1)f(u)k''(x)u_x^{(p-1)(3-\delta)+1}-\varepsilon k'(u)(1-u)^{-h}u_x^{(p-1)(2-\delta)} \\
& -\{[(5p-2\delta p+2\delta-6)k'(u)-(2p-\delta p+\delta-3)\tfrac{k(u)}{f(u)}f'(u)](1-u) \\
& +[(p-1)(2-\delta)-1]hk(u)\}\varepsilon u_x^{(p-1)(2-\delta)}(1-u)^{-h-1}.
\end{aligned}
\tag{3.6}
$$

Since $(\widetilde{H_3})$ hold, we have

$$k(u)<0, k'(u)>0, k''(u)>0, f(u)>0, f'(u)<0.$$

Since $\delta \le 2-1/(p-1)$, so

$$
\begin{aligned}
& 2p-\delta p+\delta-3 \ge 0, \\
& 5p-2\delta p+2\delta-6 \ge 0.
\end{aligned}
\tag{3.7}
$$

Next, we prove that $Q(x,t)\le 0$. We only need to prove the last term of (3.6) is negative. Since

$$
\begin{aligned}
& [(5p-2\delta p+2\delta-6)k'(u)-(2p-\delta p+\delta-3)\tfrac{k(u)}{f(u)}f'(u)](1-u)+[(p-1)(2-\delta)-1]hk(u) \\
= {} & -qf'(u)u^{-q(p-1)(\delta-1)-q-1}(3p-\delta p+\delta-3)(1-u)+qf(u)\{[q(p-1)(\delta-1)+q+1] \\
& (5p-2\delta p+2\delta-6)u^{-q(p-1)(\delta-1)-q-2}(1-u)-h[(p-1)(2-\delta)-1]u^{-q(p-1)(\delta-1)-q-1}\} \\
\ge {} & qf(u)\{[q(p-1)(\delta-1)+q+1](5p-2\delta p+2\delta-6)u^{-q(p-1)(\delta-1)-q-2}(1-u) \\
& -h[(p-1)(2-\delta)-1]u^{-q(p-1)(\delta-1)-q-1}\} \\
\ge {} & 0,
\end{aligned}
$$

if τ is sufficiently close to T.

So we can conclude that $Q(x,t)\le 0$, it means

$$\psi_t-(p-1)f(u)u_x^{p-2}\psi_{xx}-(p-1)(p-2)f(u)u_x^{p-3}u_{xx}\psi_x-J(x,t)\psi\le 0,$$

for $(x,t)\in(0,T-\tau)\times(\tau,T)$.

In addition, on the parabolic boundary, since $x=0$ is the only quenching point if ε is enough small, then

$$
\begin{aligned}
& \psi(T-\tau,t)<0, \quad t\in(\tau,T), \\
& \psi(x,\tau)<0, \qquad x\in(0,T-\tau).
\end{aligned}
$$

At $x=0$, we have

$$
\begin{aligned}
\psi_x(0,t) = {} & -\varepsilon q\{[\varepsilon q(2-\delta)+q(p-1)(\delta-1)+1]u^{-qp-1}+(2-\delta)(1-u(0,t))^{-h}\}u^{-q-1}(0,t) \\
& +q[\varepsilon(2-\delta)-1]u^{-q-1}(0,t)\psi(0,t) \\
\le {} & q[\varepsilon(2-\delta)-1]u^{-q-1}(0,t)\psi(0,t),
\end{aligned}
$$

provided ε is enough small and $\tau \to T$.

Hence, by the maximum principle, we have $\psi(x,t) \le 0$ in $[0, T-\tau] \times [\tau, T)$. In particular, $\psi(0,t) \le 0$, which implies

$$u_t(0,t) \le \varepsilon k(u) u_x^{(p-1)(2-\delta)}(0,t) = -\varepsilon q f(u) u^{-qp-1}(0,t). \tag{3.8}$$

Integrating (3.8) with respect to time from t to T, it gives

$$\int_0^{u(0,t)} \frac{u^{qp+1}}{f(s)} ds \ge C_2(T-t),$$

where $C_2 = \varepsilon q$. The proof of Theorem 3.2 is complete. □

Corollary 3.1. Assume that $(H_1), (H_2)$ and (H_3) hold, then the solution of the problem (1.1)satisfies

$$C_2 q(T-t) \le \int_0^{u(0,t)} \frac{s^{qp+1}}{f(s)} ds \le C_1 q(T-t),$$

for t sufficiently close to T, where C_1, C_2 are positive constant which are given in Theorem 2.2 and Theorem 2.3.

4 Conclusion

In the paper, we proved that the solution of problem (1.1) quenches in finite time and gave the corresponding quenching rate of the solution.

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Competing Interests

Authors have declared that no competing interests exist.

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