



On Parametric Multi-level Multi-objective Fractional Programming Problems with Fuzziness in the Constraints

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Authors' contributions

This work was carried out in collaboration between all authors. Author MSO designed the study, wrote the protocol and supervised the work. Authors OEE and MAES carried out all computational work and performed the stability analysis. Authors OEE and MAES managed the analyses of the study. Author MAES wrote the first draft of the manuscript. Authors MSO and OEE managed the literature searches and edited the manuscript. All authors read and approved the final manuscript.

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Abstract

In this paper, a new concept of the fuzzy stability set of the first kind for multi-level multi-objective fractional programming (ML-MOFP) problems having a single-scalar parameter in the objective functions and fuzziness in the right-hand side of the constraints has been introduced. Firstly, A parametric ML-MOFP model with crisp set of constraints is established based on the α -cut approach. Secondly, a fuzzy goal programming (FGP) approach is used to find an α -Pareto optimal solution of the parametric ML-MOFP problem. Thus, the FGP approach is used to achieve the highest degree of each membership goal by minimizing the sum of the negative deviational variables. Finally, the fuzzy stability set of the first kind corresponding to the obtained α -Pareto optimal solution is developed here, by extending the Karush-Kuhn-Tucker optimality conditions of multi-objective programming problems. An algorithm to clarify the developed fuzzy stability set of the first for parametric ML-MOFP problem as well as Illustrative numerical example are presented.

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1 Introduction

Hierarchical optimization or multi-level mathematical programming (ML-MP) techniques are extensions of Stackleberg games for solving decentralized planning problems with multiple decision makers (DMs) in a hierarchical organization [1]. The basic concept of multi-level programming technique is that the first-level decision maker (FLDM) sets his/her goal and/or decision, and then asks each subordinate level of the organization for their optima, that calculated in isolation. The lower level decision makers' decisions are then submitted and modified by the FLDM in consideration of the overall benefit for the organization the process continues until a satisfactory solution is reached [2,3]. ML-MP are common in government policies, competitive economic systems, supply chains, vehicle path planning problems, and so on [4]. During the past few decades, ML-MP [1,2,5] have been deeply studied and many methodologies have been developed for solving such problems. Baky [3] studied FGP algorithm for solving a decentralized bi-level multi-objective programming problem.

The solution of bi-level large scale quadratic programming problem with stochastic parameters in the constraints has been studied by Emam et al. [6]. Saad et al. [7] presented a method for solving a three-level quadratic programming problem where some or all of its coefficients in the objective function are rough intervals. Pramanik and Roy [1] adopted fuzzy goals to specify the decision variables of higher level DMs and proposed weighted/ unweighted FGP models for solving ML-MP to obtain a satisfactory solution. Emam applied an interactive approach on bi-level integer multi-objective fractional programming problem [8]. Multi-level decision-making problems were recently studied by Chen and Chen [9].

Fractional optimization problem is one of the most difficult problems in the field of optimization. Optimization of the ratio of two functions is called fractional programming (ratio optimization) problem [10]. Indeed, in such situations, it is often a question of optimizing a ratio of output/employee, profit/cost, inventory/sales, student/cost, doctor/patient, and so on subject to some constraints [11]. Such type of problems in large hierarchical organizations of complex and conflicting multi-objectives formulate ML-MOFP problems. Omran et al. [12] extended the fuzzy approach to solve a three-level fractional programming problem with rough coefficient in the constraints.

In real world decision-making situations, mathematical programming models involving fuzzy parameters were viewed to be more realistic versions than the conventional one [9]. Therefore, the parameters involved in the right-hand side of the constraints of the parametric ML-MOFP problem are assumed to be characterized by fuzzy numbers.

Osman [13] introduced the notions of the solvability set, stability set of the first kind (the set of all parameters corresponding to the efficient solution) and stability set of the second kind and analyzed these concepts for parametric convex programming. Stability of multi-objective non-linear programming problems with fuzzy parameters in the constraints was studied by Kassem and Ammar [14]. Saad [15] presented stability of proper efficient solutions in multi-objective fractional programming problems under fuzziness. Saad and Hughes [16], considered bicriterion integer linear fractional programs with single-scalar parameter in the objective functions. Recently, a parametric study on multi-objective integer quadratic programming problems under uncertainty has been presented by Emam [17].

Parametric programming investigates the effect of predetermined continuous variations in the objective function coefficients and the right-hand side of the constraints on the optimum solution [18]. In parametric analysis the objective function and the right-hand side vectors are replaced with parameterized function $c(\theta)$ and $b(\alpha)$, where θ and α are the parameter of variation. The general idea of parametric analysis is to

start with the α -Pareto optimal solution at $\theta = \theta^0$ and $\alpha = \alpha^*$. Then by utilizing the Karush-Kuhn-Tucker (KKT) optimality conditions the fuzzy stability set of the first kind (the set of parameters for which the solution at $\theta = \theta^0$ and $\alpha = \alpha^*$ remain optimal and feasible) is determined [19].

Different basic notions like solvability set and stability set of the first kind for parametric multi-objective programming have been studied in several papers. In the present research, these notions have been extended to introduce the fuzzy stability set of the first kind for parametric fuzzy ML-MOFP problems. The proposed parametric fuzzy ML-MOFP problem involves a single-scalar parameter in the objective functions and fuzzy parameters in the right-hand side of the constraints. Firstly, a numerical parametric ML-MOFP model is established based on a confidence level (α -level) then, a FGP approach is considered for finding an α -Pareto optimal solution for such problem. In FGP approach, the membership functions for the defined fuzzy goals are developed. Also, in the proposed approach, membership goals of the objective functions are linearized. Then, the highest degree of each membership goals is achieved by minimizing the sum of the negative deviational variables. Secondly, after obtaining an α -Pareto optimal solution, the parametric FGP model is set up. Thus, based on the Kuhn-Tucker optimality conditions for multi-objective programming problems, we apply KKT conditions on the parametric FGP model of the parametric fuzzy ML-MOFP problem to formulate a system of equations. Then, the fuzzy stability set of the first kind, obtained from the reduced system of equations.

The rest of this paper is organized as follows. Section 2 presents the parametric fuzzy ML-MOFP problems formulation and introduces its solution concepts. Section 3 explains the developed FGP approach for solving such problems. Section 4 proposes the fuzzy stability set of the first kind for parametric fuzzy ML-MOFP problems. An algorithm for obtaining the fuzzy stability set of the first kind for parametric fuzzy ML-MOFP problems is introduced in section 5. Illustrative example is given in section 6. This paper ends with some concluding remarks in section 7.

2 Problem Formulation and Solution Concepts

Multi-level programming problems have more than one decision maker (DM). A decision maker is located at each decision level and a vector of fractional objective functions, with single-scalar parameter θ , need to be optimized. Consider the hierarchical system be composed of a t -level decision makers. Let the decision maker at the i^{th} -level denoted by DM_i controls over the decision variable $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{in_i}) \in R^{n_i}, i = 1, 2, \dots, t$. where $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_t) \in R^n$ and $n = \sum_{i=1}^t n_i$ and furthermore assumed that

$$F_i(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_t, \theta) \equiv F_i(\mathbf{x}, \theta): R^{n_1} \times R^{n_2} \times \dots \times R^{n_t} \rightarrow R^{m_i}, \quad i = 1, 2, \dots, t, \quad (1)$$

are the vector of fractional objective functions with single-scalar parameter $\theta \in R$ for $DM_i, i = 1, 2, \dots, t$. Mathematically, parametric fuzzy ML-MOFP problem may be formulated as follows [1,2,5]:

[1st Level]

$$\max_{\mathbf{x}_1} F_1(\mathbf{x}, \theta) = \max_{\mathbf{x}_1} (f_{11}(\mathbf{x}, \theta), f_{12}(\mathbf{x}, \theta), \dots, f_{1k_1}(\mathbf{x}, \theta)), \quad (2)$$

where $\mathbf{x}_2, \mathbf{x}_3, \dots, \mathbf{x}_t$ solves

[2nd Level]

$$\max_{\mathbf{x}_2} F_2(\mathbf{x}, \theta) = \max_{\mathbf{x}_2} (f_{21}(\mathbf{x}, \theta), f_{22}(\mathbf{x}, \theta), \dots, f_{2k_2}(\mathbf{x}, \theta)), \quad (3)$$

⋮

where \mathbf{x}_t solves

[t^{th} Level]

$$\max_{x_t} F_t(x, \theta) = \max_{x_t} (f_{t1}(x, \theta), f_{t2}(x, \theta), \dots, f_{tk_t}(x, \theta)), \quad (4)$$

subject to

$$x \in G(x, \tilde{b}) = \left\{ x \in R^n \left| \sum_{j=1}^n A_{ij} x_j \leq \tilde{b}_i, x_j \geq 0, (i = 1, 2, \dots, m) \right. \right\}, \quad (5)$$

where

$$f_{ij}(x, \theta) = \frac{N_{ij}(x, \theta)}{D_{ij}(x)} = \frac{(c^{ij} + h^{ij}\theta)x + \alpha^{ij}}{d^{ij}x + \beta^{ij}}, \quad i = 1, 2, \dots, t, \quad j = 1, 2, \dots, k_i. \quad (6)$$

also $c^{ij}, h^{ij}, d^{ij} \in R^n$, and α^{ij}, β^{ij} are scalars in addition to that \tilde{b} is an m -vector of fuzzy number characterized by any type of membership functions, such as triangular, trapezoidal, depending on DM's preference. A_i are the matrices of size $m \times n_i$, $i = 1, 2, \dots, t$. It is customary to assume that $D_{ij}(x) > 0 \forall x \in G(x, \tilde{b})$, and represents the multi-level convex constraints feasible choice set in the fuzzy environment.

Definition 1 [19]. Let \tilde{b} be a fuzzy subset of R with membership function $\mu_{\tilde{b}}$. It is said that \tilde{b} is a fuzzy number if the following conditions are satisfied:

- \tilde{b} is normal, i.e., there exists an $x \in R$ such that $\mu_{\tilde{b}}(x) = 1$,
- $\mu_{\tilde{b}}$ is quasi-concave, i.e., $\mu_{\tilde{b}}(wx + (1-w)y) \geq \min\{\mu_{\tilde{b}}(x), \mu_{\tilde{b}}(y)\}$ for all $w \in [0, 1]$,
- $\mu_{\tilde{b}}$ is upper semicontinuous, i.e., $\{x: \mu_{\tilde{b}}(x) \geq \alpha\}$ is a closed subset of U for all $\alpha \in [0, 1]$,
- The 0-level set \tilde{b}_0 is a compact subset of U .

Definition 2 [14]. The α -level set of the vector of fuzzy parameters \tilde{b} , is defined as an ordinary set $L_\alpha(\tilde{b})$ for which the degree of its membership function exceeds the level set $\alpha \in [0, 1]$, where:

$$L_\alpha(\tilde{b}) = \{b \in R^m | \mu_{\tilde{b}}(x) \geq \alpha\} = \{b \in [(b)_\alpha^L, (b)_\alpha^U] | \mu_{\tilde{b}} \geq \alpha, \}$$

Based on the parametric fuzzy ML-MOFP model (2) – (5), with single-scalar parameter $\theta \in R$ in the objective functions and fuzzy parameters in the right hand side of the constraints. Let $\mu_{\tilde{b}}$, be the membership functions which represents the fuzziness in the corresponding vector \tilde{b} . Thus, for a specified value of $\alpha = \alpha^* \in [0, 1]$, estimated by all DM, the parametric α -(ML-MOFP) problem reformulated as follow:

[1st Level]

$$\max_{x_1} F_1(x, \theta) = \max_{x_1} (f_{11}(x, \theta), f_{12}(x, \theta), \dots, f_{1k_1}(x, \theta)), \quad (7)$$

where x_2, x_3, \dots, x_t solves

[2nd Level]

$$\max_{x_2} F_2(x, \theta) = \max_{x_2} (f_{21}(x, \theta), f_{22}(x, \theta), \dots, f_{2k_2}(x, \theta)), \quad (8)$$

$$\vdots$$

where x_t solves

[t^{th} Level]

$$\max_{x_t} F_t(x, \theta) = \max_{x_t} (f_{t1}(x, \theta), f_{t2}(x, \theta), \dots, f_{tk_t}(x, \theta)), \quad (9)$$

subject to

$$x \in G_\alpha(x, \mathbf{b}) = \left\{ x \in R^n \left| \sum_{j=1}^n A_{ij} x_j \leq b_l, \quad x_j \geq 0, b_l \in \mu_{\bar{b}_l}(x) \geq \alpha, l = 1, 2, \dots, m. \right. \right\}, \quad (10)$$

where the crisp system of constraints, in equation (10), at an α -level denoted by G_α which form a compact set.

Definition 3. For any $x_1(x_1 \in (G_1)_\alpha = \{x_1 | x = (x_1, x_2, \dots, x_t) \in (G)_\alpha\})$ given by FLDM and $x_2(x_2 \in (G_2)_\alpha = \{x_2 | x = (x_1, x_2, \dots, x_t) \in (G)_\alpha\})$ given by SLDM, if the decision variable $x_t(x_t \in (G_t)_\alpha = \{x_t | x = (x_1, x_2, \dots, x_t) \in (G)_\alpha\})$ is the α -Pareto optimal solution of the TLDM, then (x_1, x_2, \dots, x_t) is an α -feasible solution of the parametric α -(ML-MOFP) problem.

Definition 4. If $x^* = (x_1^*, x_2^*, \dots, x_t^*)$ is an α -feasible solution of the parametric α -(ML-MOFP) problem; no other α -feasible solution $x = (x_1, \dots, x_t) \in G_\alpha$ exist, such that $f_{1j}(x^*, \theta^0) \leq f_{1j}(x, \theta^0)$ with at least one strict inequality hold for $j(j = 1, 2, \dots, k_1)$; so $(x_1^*, x_2^*, \dots, x_t^*)$ is the α -Pareto optimal solution of the parametric α -(ML-MOFP) problem.

Assuming that the parametric α -(ML-MOFP) problem has an α -Pareto optimal solution x^* at θ^0 .

3 Fuzzy Goal Programming Approach of Parametric Fuzzy ML-MOFP Problems

In the proposed FGP approach for parametric α -(ML-MOFP) in order to obtain the compromise (satisfactory) solution that is an α -Pareto optimal solution. The vector of objective functions for each DM is formulated as a fuzzy goal characterized by its' membership function $\mu_{(f_{ij}(x, \theta^0))}$ ($i = 1, 2, \dots, t$), ($j = 1, 2, \dots, k_i$) [1,2]. The model formulation and solution process are carried out at $\theta = \theta^0$.

3.1 Characterization of membership functions

To define the membership functions of the fuzzy goals [3], each objective function's individual maximum is taken as the corresponding aspiration level, as follows:

$$u_{ij}^0 = \max_{x \in G_\alpha} f_{ij}(x, \theta^0), \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (11)$$

where u_{ij}^0 , give the upper tolerance limit or aspired level of achievement for the membership function of ij^{th} objective function at $\theta = \theta^0$. Similarly, each objective function's individual minimum is taken as the corresponding aspiration level, as follows:

$$g_{ij}^0 = \min_{x \in G_\alpha} f_{ij}(x, \theta^0), \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (12)$$

where g_{ij}^0 , give the lower tolerance limit or lowest acceptable level of achievement for the membership function of i_j^{th} objective function. Assuming that the values of $f_{ij}(\mathbf{x}, \theta^0) \geq u_{ij}^0$, ($i = 1, 2, \dots, t$), ($j = 1, 2, \dots, k_i$), are acceptable and all values $f_{ij}(\mathbf{x}, \theta^0) \leq g_{ij}^0$, are absolutely unacceptable. And all values $g_{ij}^0 \leq f_{ij}(\mathbf{x}, \theta^0) \leq u_{ij}^0$ described by the membership function $\mu_{(f_{ij}(\mathbf{x}, \theta^0))} = \mu_1$, as shown in Fig. (1), for the i_j^{th} fuzzy goal [1]:

$$\mu_1 = \begin{cases} 1, & \text{if } f_{ij}(\mathbf{x}, \theta^0) \geq u_{ij}^0, \\ \frac{f_{ij}(\mathbf{x}, \theta^0) - g_{ij}^0}{u_{ij}^0 - g_{ij}^0}, & \text{if } g_{ij}^0 \leq f_{ij}(\mathbf{x}, \theta^0) \leq u_{ij}^0, \quad (i = 1, \dots, t), (j = 1, \dots, k_i), \\ 0, & \text{if } f_{ij}(\mathbf{x}, \theta^0) \leq g_{ij}^0, \end{cases} \quad (13)$$

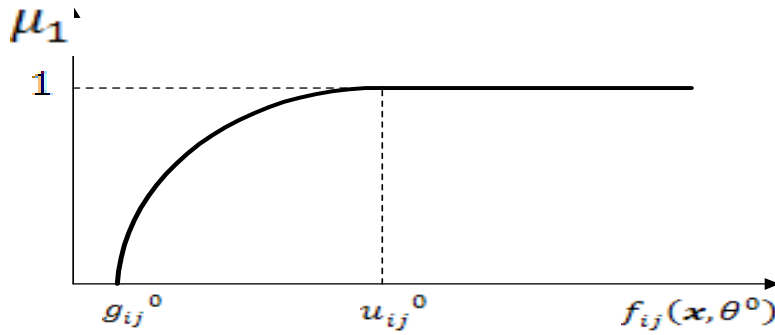


Fig. 1. Membership functions of maximization type for $f_{ij}(\mathbf{x}, \theta^0)$

Following the basic concept of multi-level programming problems the first level decision maker sets his/her goals and/or decisions and then asks subordinate level for their optima [1-3]. Therefore, to study the fuzzy stability set of the first kind the vector of decision variables x_{ik} , ($i = 1, 2, \dots, t - 1$), ($k = 1, 2, \dots, n_i$) for the top levels are taken as binding constraints for the t^{th} -level problem as follows:

$$x_{ik} = x_{ik}^* \quad (i = 1, 2, \dots, t - 1), \quad (k = 1, 2, \dots, n_i), \quad (14)$$

3.2 Fuzzy goal programming methodology

In the decision-making context, each decision maker is interested in maximizing his or her own objective function; the optimal solution of each DM, when calculated in isolation, would be considered as the best solution and the associated value of the objective function can be considered as the aspiration level of the corresponding fuzzy goal [2]. In fuzzy programming approach, the highest degree of membership is one. So, for the defined membership function in equation (13), the flexible membership goals having the aspired level unity can be represented as follows [20]:

$$\mu_{f_{ij}}(f_{ij}(\mathbf{x}, \theta^0)) + d_{ij}^- - d_{ij}^+ = 1, \quad (i = 1, 2, \dots, t), \quad (j = 1, 2, \dots, k_i), \quad (15)$$

or equivalently as:

$$\frac{f_{ij}(\mathbf{x}, \theta^0) - g_{ij}^0}{u_{ij}^0 - g_{ij}^0} + d_{ij}^- - d_{ij}^+ = 1, \quad (i = 1, 2, \dots, t), \quad (j = 1, 2, \dots, k_i), \quad (16)$$

where $d_{ij}^-, d_{ij}^+ \geq 0$ with $d_{ij}^- \times d_{ij}^+ = 0$, represent the under- and over- deviations, respectively, from the aspired levels [1,3,4].

In the classical methodology of goal programming, the under- and over- deviational variables are included in the achievement function for minimizing them depends upon the type of the objective functions to be optimized [1,3]. Thus, considering the goal achievement problem at the same priority level, the equivalent proposed FGP model of the parametric α -(ML-MOFP) problem can be formulated as follows:

$$\min Z = \sum_{j=1}^{k_1} w_{1j}^- d_{1j}^- + \sum_{j=1}^{k_2} w_{2j}^- d_{2j}^- + \dots + \sum_{j=1}^{k_t} w_{tj}^- d_{tj}^-, \quad (17)$$

subject to

$$\frac{f_{ij}(\mathbf{x}, \theta^0) - g_{ij}^0}{u_{ij}^0 - g_{ij}^0} + d_{ij}^- - d_{ij}^+ = 1, \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (18)$$

$$\mathbf{x}_{iv} = \mathbf{x}_{iv}^*, \quad (i = 1, 2, \dots, t - 1), (v = 1, 2, \dots, n_i), \quad (19)$$

$$\sum_{j=1}^n A_{ij} \mathbf{x}_j \leq b_l, \quad (l = 1, 2, \dots, m), \quad \mathbf{x}_{tv} \geq 0, \quad (20)$$

$$(b_l)_\alpha^L \leq b_l \leq (b_l)_\alpha^U, \quad (l = 1, 2, \dots, m), \quad (21)$$

$$d_{ij}^- \times d_{ij}^+ = 0, \text{ and } d_{ij}^-, d_{ij}^+ \geq 0, \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (22)$$

where Z represents the achievement function consisting of the weighted under-deviational variables of the fuzzy goals. The numerical weights w_{ij}^- represent the relative importance of achieving the aspired levels of the respective fuzzy goals these values are determined as [20]:

$$w_{ij}^- = \frac{1}{u_{ij}^0 - g_{ij}^0}, \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (23)$$

3.3 Linearization of parametric membership goals

It can be easily realized that the parametric membership goals in equation (13) are inherently non-linear in nature and this may create computational difficulties in the solution process. To avoid such problems, a linearization procedure is presented in this section. Following Pal et al. [11] the parametric ij^{th} membership goals with single-scalar parameter θ can be presented as:

$$\mu_{f_{ij}}(f_{ij}(\mathbf{x}, \theta^0)) + d_{ij}^- - d_{ij}^+ = 1, \quad (i = 1, 2, \dots, t), (j = 1, \dots, k_i), \quad (24)$$

$$L_{ij}(f_{ij}(\mathbf{x}, \theta^0)) - L_{ij}g_{ij}^0 + d_{ij}^- - d_{ij}^+ = 1, \quad \text{where } L_{ij} = \frac{1}{u_{ij}^0 - g_{ij}^0}, \quad (25)$$

$$f_{ij}(\mathbf{x}, \theta^0) = \frac{N_{ij}(\mathbf{x}, \theta^0)}{D_{ij}(\mathbf{x})} = \frac{(\mathbf{c}^{ij} + \mathbf{h}^{ij}\theta^0) \mathbf{x} + \alpha^{ij}}{\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}}, \quad (i = 1, 2, \dots, t), (j = 1, \dots, k_i), \quad (26)$$

Considering the expression of $f_{ij}(\mathbf{x}, \theta^0)$, the above goal in equation (25) can be represented as:

$$L_{ij} \frac{(\mathbf{c}^{ij} + \mathbf{h}^{ij}\theta^0) \mathbf{x} + \alpha^{ij}}{\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}} - L_{ij}g_{ij}^0 + d_{ij}^- - d_{ij}^+ = 1, \quad (27)$$

$$\begin{aligned}
& L_{ij}[(c^{ij} + h^{ij}\theta^0) \mathbf{x} + \alpha^{ij}] - L_{ij}g_{ij}^0[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] + d_{ij}^-[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] - d_{ij}^+[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] = \\
& [\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}], \\
& L_{ij}[(c^{ij} + h^{ij}\theta^0) \mathbf{x} + \alpha^{ij}] + d_{ij}^-[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] - d_{ij}^+[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] = (1 + L_{ij}g_{ij}^0)[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}], \\
& L_{ij}[(c^{ij} + h^{ij}\theta^0) \mathbf{x} + \alpha^{ij}] + d_{ij}^-[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] - d_{ij}^+[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] = L_{ij}^0[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}],
\end{aligned}$$

where $L_{ij}^0 = (1 + L_{ij}g_{ij}^0)$,

$$\begin{aligned}
& [L_{ij}(c^{ij} + h^{ij}\theta^0) - L_{ij}^0\mathbf{d}^{ij}]\mathbf{x} + d_{ij}^-[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] - d_{ij}^+[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] = [L_{ij}^0\beta^{ij} - L_{ij}\alpha^{ij}], \\
& \mathbf{C}^{ij} \mathbf{x} + d_{ij}^-[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] - d_{ij}^+[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}] = \mathbf{G}_{ij},
\end{aligned} \tag{28}$$

Where

$$\mathbf{C}^{ij} = [L_{ij}(c^{ij} + h^{ij}\theta^0) - L_{ij}^0\mathbf{d}^{ij}] \text{ and } \mathbf{G}_{ij} = [L_{ij}^0\beta^{ij} - L_{ij}\alpha^{ij}], i = 1, \dots, t, j = 1, \dots, k_i, \tag{29}$$

Considering the method of variable change presented by Pal et al. [11], the goal expression in equation (28) can be linearized as follows. Letting $D_{ij}^- = d_{ij}^-[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}]$ and $D_{ij}^+ = d_{ij}^+[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}]$, then the linear form of expression in equation (28) is obtained as:

$$\mathbf{C}^{ij} \mathbf{x} + D_{ij}^- - D_{ij}^+ = \mathbf{G}_{ij}, \tag{30}$$

with $D_{ij}^-, D_{ij}^+ \geq 0$; and $D_{ij}^- \times D_{ij}^+ = 0$, since $d_{ij}^-, d_{ij}^+ \geq 0$, and $\mathbf{d}^{ij} \mathbf{x} + \beta^{ij} > 0$. Now, in decision making policy, minimization of d_{ij}^- means minimization of $D_{ij}^- = d_{ij}^-[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}]$ which is also non-linear. So, involvement of $d_{ij}^- \leq 1$, in the solution leads to impose the following constraint in the model of the problem:

$$\frac{D_{ij}^-}{[\mathbf{d}^{ij} \mathbf{x} + \beta^{ij}]} \leq 1. \tag{31}$$

Now, based on the simplest version of goal programming, the final proposed FGP model of the parametric α -(ML-MOFP) problem in model (17)-(22) becomes:

$$\min Z = \sum_{j=1}^{k_1} w_{1j}^- D_{1j}^- + \sum_{j=1}^{k_2} w_{2j}^- D_{2j}^- + \dots + \sum_{j=1}^{k_t} w_{tj}^- D_{tj}^-, \tag{32}$$

subject to

$$[L_{ij}(c^{ij} + h^{ij}\theta^0) - L_{ij}^0\mathbf{d}^{ij}]\mathbf{x} + D_{ij}^- - D_{ij}^+ = [L_{ij}^0\beta^{ij} - L_{ij}\alpha^{ij}] \quad \forall i, j \tag{33}$$

$$\mathbf{x}_{iv} = \mathbf{x}_{iv}^*, \quad (i = 1, 2, \dots, t-1), \quad (v = 1, 2, \dots, n_i), \tag{34}$$

$$-\mathbf{d}^{ij} \mathbf{x} + D_{ij}^- \leq \beta^{ij}, \quad (i = 1, 2, \dots, t), \quad (j = 1, 2, \dots, k_i), \tag{35}$$

$$\sum_{j=1}^n A_{ij} \mathbf{x}_j \leq b_l, \quad (l = 1, 2, \dots, m), \quad \mathbf{x}_{tv} \geq 0, \tag{36}$$

$$(b_l)_\alpha^L \leq b_l \leq (b_l)_\alpha^U, \quad (l = 1, 2, \dots, m), \tag{37}$$

$$D_{ij}^-, D_{ij}^+ \geq 0, \quad (i = 1, 2, \dots, t), \quad (j = 1, 2, \dots, k_i), \tag{38}$$

Thus, the above FGP model provides the satisfactory solution \mathbf{x}^0 for the parametric α -(ML-MOFP) problem.

4 The Fuzzy Stability Set of the First Kind for Parametric Fuzzy ML-MOFP Problem

Now, the main question is: Having solved the parametric α -(ML-MOFP) problem to what extents can its data with respect to α and θ be changed without invalidating the efficiency of its α -Pareto optimal solution (compromise solution)?

Thus the definition of the set of feasible parameters, solvability set and the fuzzy stability set of the first kind for parametric α -(ML-MOFP) problem is given as follows:

Definition 5 [15]. The set of feasible parameters for the parametric α -(ML-MOFP) problem is defined by:

$$V = \{b \in R^m \mid b_l \in L_\alpha(\bar{b}_l), \alpha \in [0,1] \ (l = 1,2, \dots, m) \text{ and } G_\alpha(\mathbf{x}, b) \neq \emptyset\}.$$

Definition 6. The solvability set B of the parametric α -(ML-MOFP) problem is defined by:

$$B = \left\{ (\theta, b) \in R \times R^m \mid \begin{array}{l} \text{parametric } \alpha - (\text{ML} - \text{MOFP}) \text{ problem} \\ \text{has an } \alpha - \text{Pareto optimal solution.} \end{array} \right\}.$$

Definition 7. Suppose that \mathbf{x}^0 be an α -Pareto optimal solution (compromise solution) of the parametric α -(ML-MOFP) problem, then the fuzzy stability set of the first kind $\mathcal{S}_1(\mathbf{x}^0, \alpha)$ corresponding to \mathbf{x}^0 is defined by:

$$\mathcal{S}_1(\mathbf{x}^0, \alpha) = \left\{ (\theta, b) \in R \times R^m \mid \begin{array}{l} \mathbf{x}^0 \text{ is an } \alpha - \text{Pareto optimal solution of} \\ \text{parametric } \alpha - (\text{ML} - \text{MOFP}) \text{ problem} \end{array} \right\}.$$

The fuzzy stability set of the first kind of the parametric α -(ML-MOFP) problem is the set of all parameters corresponding to one α -Pareto optimal solution [14,15]. It is easy to see that the fuzzy stability of the parametric α -(ML-MOFP) model (7)-(10) implies the stability of the parametric FGP model which is defined as follows:

$$\min Z = \sum_{j=1}^{k_1} w_{1j}^- D_{1j}^- + \sum_{j=1}^{k_2} w_{2j}^- D_{2j}^- + \dots + \sum_{j=1}^{k_t} w_{tj}^- D_{tj}^-, \quad (39)$$

subject to

$$[L_{ij}(\mathbf{c}^{ij} + \mathbf{h}^{ij}\theta) - L_{ij}^0 \mathbf{d}^{ij}] \mathbf{x} + D_{ij}^- - D_{ij}^+ = [L_{ij}^0 \beta^{ij} - L_{ij} \alpha^{ij}] \quad \forall i, j \quad (40)$$

$$\mathbf{x}_{iv} = \mathbf{x}_{iv}^*, \quad (i = 1, 2, \dots, t-1), \quad (v = 1, 2, \dots, n_i), \quad (41)$$

$$-\mathbf{d}^{ij} \mathbf{x} + D_{ij}^- \leq \beta^{ij}, \quad (i = 1, 2, \dots, t), \quad (j = 1, 2, \dots, k_i), \quad (42)$$

$$\sum_{j=1}^n A_{ij} \mathbf{x}_j \leq b_l, \quad (l = 1, 2, \dots, m), \quad (43)$$

$$\mathbf{x}_{tv} \geq 0, \quad (v = 1, 2, \dots, n_t), \quad (44)$$

$$(b_l)_\alpha^L \leq b_l \leq (b_l)_\alpha^U, \quad (l = 1, 2, \dots, m), \quad (45)$$

$$D_{ij}^-, D_{ij}^+ \geq 0, \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (46)$$

4.1 Utilization of the Karush-Kuhn-Tucker optimality conditions corresponding to parametric FGP model

The Lagrangean function of parametric FGP model (39)-(46) follows as [16,19]:

$$\begin{aligned} L = & \left[\sum_{j=1}^{k_1} w_{1j}^- D_{1j}^- + \sum_{j=1}^{k_2} w_{2j}^- D_{2j}^- + \dots + \sum_{j=1}^{k_t} w_{tj}^- D_{tj}^- \right] \\ & + \lambda_{ij} \left[L_{ij}(\mathbf{c}^{ij} + \mathbf{h}^{ij}\theta) - L_{ij}^0 \mathbf{d}^{ij} \right] \mathbf{x} + D_{ij}^- - D_{ij}^+ - \left[L_{ij}^0 \beta^{ij} - L_{ij} \alpha^{ij} \right] + \xi_{iv} [\mathbf{x}_{iv} - \mathbf{x}_{iv}^*] \\ & - \psi_{tv} \mathbf{x}_{tv} + \mu_{ij} [-\mathbf{d}^{ij} \mathbf{x} + D_{ij}^- - \beta^{ij}] + v_l \left[\sum_{j=1}^n A_{ij} x_j - b_l \right] + \eta_l [b_l - (b_l)_\alpha^U] \\ & + \phi_l [-b_l + (b_l)_\alpha^L] + \gamma_{ij} [-D_{ij}^-] + \delta_{ij} [-D_{ij}^+] \end{aligned} \quad (47)$$

where $\lambda, \xi, \psi, \mu, v, \eta, \phi, \gamma$ and δ are the Lagrange multipliers. Then the Karush-Kuhn-Tucker necessary optimality conditions [16,19] corresponding to the parametric FGP model (39)-(45), which has the above Lagrangean function, will have the following form:

$$\frac{\partial L}{\partial \mathbf{x}_j} = \lambda_{ij} [L_{ij}(\mathbf{c}^{ij} + \mathbf{h}^{ij}\theta) - L_{ij}^0 \mathbf{d}^{ij}] + \xi_{iv} - \psi_{tv} - \mu_{ij} \mathbf{d}^{ij} - \sum_{i=1}^m v_i A_{ij} = 0, \quad (j = 1, 2, \dots, n), \quad (48)$$

$$\frac{\partial L}{\partial b_l} = -v_l + \eta_l - \phi_l = 0, \quad (l = 1, 2, \dots, m), \quad (49)$$

$$\frac{\partial L}{\partial D_{ij}^-} = w_{ij}^- + \lambda_{ij} + \mu_{ij} - \gamma_{ij} = 0, \quad (i = 1, 2, \dots, t), \quad (j = 1, 2, \dots, k_i), \quad (50)$$

$$\frac{\partial L}{\partial D_{ij}^+} = -\lambda_{ij} - \delta_{ij} = 0, \quad (i = 1, 2, \dots, t), \quad (j = 1, 2, \dots, k_i), \quad (51)$$

$$[L_{ij}(\mathbf{c}^{ij} + \mathbf{h}^{ij}\theta) - L_{ij}^0 \mathbf{d}^{ij}] \mathbf{x} + D_{ij}^- - D_{ij}^+ - [L_{ij}^0 \beta^{ij} - L_{ij} \alpha^{ij}] = 0, \quad \forall i, j \quad (52)$$

$$\mathbf{x}_{iv} - \mathbf{x}_{iv}^* = 0, \quad (i = 1, 2, \dots, t-1), \quad (v = 1, 2, \dots, n_i), \quad (53)$$

$$-\mathbf{d}^{ij} \mathbf{x} + D_{ij}^- - \beta^{ij} \leq 0, \quad (i = 1, 2, \dots, t), \quad (j = 1, 2, \dots, k_i), \quad (54)$$

$$\sum_{j=1}^n A_{ij} x_j - b_l \leq 0, \quad (l = 1, 2, \dots, m), \quad (55)$$

$$(b_l)_\alpha^L - b_l \leq 0, \quad (l = 1, 2, \dots, m), \quad (56)$$

$$b_l - (b_l)_\alpha^U \leq 0, \quad (l = 1, 2, \dots, m), \quad (57)$$

$$D_{ij}^-, D_{ij}^+ \geq 0, \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (58)$$

$$\mathbf{x}_{tv} \geq 0, \quad (v = 1, 2, \dots, n_t), \quad (59)$$

$$\mu_{ij}[-\mathbf{d}^{ij} \mathbf{x} + D_{ij}^- - \beta^{ij}] = 0, \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (60)$$

$$v_l \left[\sum_{j=1}^n A_{ij} x_j - b_l \right] = 0, \quad (l = 1, 2, \dots, m), \quad (61)$$

$$\eta_l [b_l - (b_l)_\alpha^U] = 0, \quad (l = 1, 2, \dots, m), \quad (62)$$

$$\phi_l [-b_l + (b_l)_\alpha^L] = 0, \quad (l = 1, 2, \dots, m), \quad (63)$$

$$\gamma_{ij} D_{ij}^- = 0, \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (64)$$

$$\delta_{ij} D_{ij}^+ = 0, \quad (i = 1, 2, \dots, t), (j = 1, 2, \dots, k_i), \quad (65)$$

$$\psi_{tv} \mathbf{x}_{tv} = 0, \quad (v = 1, 2, \dots, n_t), \quad (66)$$

$$\psi, \mu, v, \eta, \phi, \gamma, \delta \geq 0, \quad \text{and} \quad \lambda, \xi \in R, \quad (67)$$

where all the expressions of the Kuhn-Tucker conditions (47)-(67) are evaluated at the α -Pareto optimal solution \mathbf{x}^0 of the parametric FGP model. Moreover, $\xi, \psi, \mu, v, \eta, \phi, \gamma$ and δ are the Lagrange multipliers. Solving the system of equations (47)-(67), the fuzzy stability set of the first kind $\mathbf{S}_1(\mathbf{x}^0, \alpha)$ for parametric fuzzy multi-level multi-objective fractional programming problem with single-scalar parameter in the objective functions and fuzziness in constraints will be obtained.

5 Algorithm for Determination of the Fuzzy Stability Set of the First Kind $\mathbf{S}_1(\mathbf{x}^0, \alpha)$

Following the above discussion, an algorithm will be developed for obtaining the fuzzy stability set of the first kind $\mathbf{S}_1(\mathbf{x}^0, \alpha)$ for parametric fuzzy ML-MOFP problem as follows:

Stage I: obtain a compromise solution of the problem

- Step 1.** Set the value of α , acceptable for all decision makers.
- Step 2.** Postulate that $\theta = \theta^0$ at the first.
- Step 3.** Compute the individual maximum and minimum values for each objective function.
- Step 4.** Set the goals and the upper tolerance limits for each objective function in all levels, according to equations (11)-(12).
- Step 5.** Evaluate the weights w_{ij}^- as defined in equation (23).
- Step 6.** Set $\ell = 1$, for the i^{th} level problem.
- Step 7.** Formulate the membership functions $\mu_1 = \mu_{f_{ij}}(f_{ij}(\mathbf{x}, \theta^0))$ $j = 1, 2, \dots, k_\ell$, as in equation (13).
- Step 8.** Do the linearization procedures for each parametric membership goal according to equations (28)-(30) at $\theta = \theta^0$.
- Step 9.** Solve the ℓ^{th} level FGP model to get the decision variables $\mathbf{x}_{\ell v} = \mathbf{x}_{\ell v}^*$.
- Step 10.** If $\ell > t - 1$, then go to the Step 11; otherwise set $\ell = \ell + 1$, and go to Step 7.
- Step 11.** Solve the final FGP model, as in equations (32)-(38). to get the α -Pareto optimal solution \mathbf{x}^0 .

Stage II: determination of the fuzzy stability set of the first kind $S_1(x^0, \alpha)$

- Step 12.** Formulate the parametric FGP model (39)-(46).
Step 13. Obtain the Lagrangian function, for the final FGP model, as in equation (47).
Step 14. Apply the Kuhn-Tucker optimality conditions to find, the fuzzy stability set of the first kind, equations (48)-(67).
Step 15. Reduce the system of equations (48)-(67), to obtain the fuzzy stability set of the first kind $S_1(x^0, \alpha)$ and Stop.

6 Illustrative Example

To demonstrate the proposed algorithm for finding the fuzzy stability set of the first kind, consider the following parametric fuzzy ML-MOFP problem with single-scalar parameter in the objective functions and fuzziness in the right hand side of the constraints.

[1st Level]

$$\max_{x_1} \left(f_{11}(x, \theta) = \frac{2\theta x_1 + x_2 + (1 - \theta)x_3 + 5}{x_1 + 2x_2 + 4x_3}, f_{12}(x, \theta) = \frac{6x_1 + (4 - \theta)x_2 - \theta x_3}{x_1 + 2x_2 + x_3} \right),$$

where x_2, x_3 solves

[2nd Level]

$$\max_{x_2} \left(f_{21}(x, \theta) = \frac{(2 - \theta)x_1 + 3\theta x_2 - 2x_3 + 4}{2x_1 + x_2 + x_3}, f_{22}(x, \theta) = \frac{3\theta x_1 - (3 + \theta)x_2 + 4x_3 + 1}{x_1 + 2x_2 + x_3} \right),$$

where x_3 solves

[3rd Level]

$$\max_{x_3} \left(f_{31}(x, \theta) = \frac{7x_1 + (\theta - 1)x_2 - 3x_3}{x_1 + 3x_2 + 2x_3}, f_{32}(x, \theta) = \frac{6\theta x_1 - (2 + \theta)x_2 + 3x_3 + 2}{x_1 + 3x_2} \right),$$

subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &\leq \tilde{b}_1, \\ x_1 - 2x_2 + 3x_3 &\leq \tilde{b}_2, \\ x_1 + 2x_2 &\geq 3, \\ x_1 \geq 0, x_2 \geq 0, x_3 &\geq 0. \end{aligned}$$

where, \tilde{b}_1 and \tilde{b}_2 , are fuzzy parameters and are characterized by the following triangular fuzzy numbers: $\tilde{b}_1 = (3, 10, 15)$, $\tilde{b}_2 = (2, 7, 12)$.

Stage I: finding the α -compromise solution of the parametric fuzzy ML-MOFP problem.

For a desired value of α , assume that an α -level of 0.2 is accepted by the three level DMs then we get: $3 + 7\alpha \leq b_1 \leq 15 - 5\alpha$ and $2 + 5\alpha \leq b_2 \leq 12 - 5\alpha$, choosing $b_1 = 10$ and $b_2 = 8$. Then assuming that the parametric α -(ML-MOFP) problem has an α -Pareto optimal solution x^0 at $\theta = \theta^0 = 1$.

[1st Level]

$$\max_{x_1} \left(f_{11}(x) = \frac{2x_1 + x_2 + 5}{x_1 + 2x_2 + 4x_3}, f_{12}(x) = \frac{6x_1 + 3x_2 - x_3}{x_1 + 2x_2 + x_3} \right),$$

where x_2, x_3 solves

[2nd Level]

$$\max_{x_2} \left(f_{21}(x) = \frac{x_1 + 3x_2 - 2x_3 + 4}{2x_1 + x_2 + x_3}, f_{22}(x) = \frac{3x_1 - 4x_2 + 4x_3 + 1}{x_1 + 2x_2 + x_3} \right),$$

where x_3 solves

[3rd Level]

$$\max_{x_3} \left(f_{31}(x) = \frac{7x_1 - 3x_3}{x_1 + 3x_2 + 2x_3}, f_{32}(x) = \frac{6x_1 - 3x_2 + 3x_3 + 2}{x_1 + 3x_2} \right),$$

subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &\leq 10, \\ x_1 - 2x_2 + 3x_3 &\leq 8, \\ x_1 + 2x_2 &\geq 3, \\ x_1 \geq 0, x_2 \geq 0, x_3 &\geq 0. \end{aligned}$$

The individual maximum and minimum values are summarized in Table 1. The decided aspiration levels, upper tolerance limits and the weights w_{ij} are also given.

Table 1. Individual maximum, minimum values, u_{ij}, g_{ij} and weights w_{ij}

	$f_{11}(x)$	$f_{12}(x)$	$f_{21}(x)$	$f_{22}(x)$	$f_{31}(x)$	$f_{32}(x)$
$\max (f_{ij}(x))$	3.667	6	5.667	3.57	7	8.33
$\min (f_{ij}(x))$	0.3	0.125	0.226	-1.95	-0.93	-0.93
u_{ij}	3.7	6	5.7	3.6	7	8
g_{ij}	0.3	0.125	0.23	-1.95	-0.93	-0.93
w_{ij}	0.294	0.17	0.183	0.18	0.126	0.112

The coefficient of the linearized membership goals are presented in Table 2.

Table 2. The coefficient of the linearized membership goals $(C^{ij})^T$ and G_{ij}

	$f_{11}(x)$	$f_{12}(x)$	$f_{21}(x)$	$f_{22}(x)$	$f_{31}(x)$	$f_{32}(x)$
$(C^{ij})^T$	$\begin{pmatrix} -0.512 \\ -1.91 \\ -4.4 \end{pmatrix}^T$	$\begin{pmatrix} 0 \\ -1.53 \\ -1.19 \end{pmatrix}^T$	$\begin{pmatrix} -1.897 \\ -0.491 \\ -1.41 \end{pmatrix}^T$	$\begin{pmatrix} -0.109 \\ -2.018 \\ 0.071 \end{pmatrix}^T$	$\begin{pmatrix} 0 \\ -2.64 \\ -2.138 \end{pmatrix}^T$	$\begin{pmatrix} -0.224 \\ -3.024 \\ 0.336 \end{pmatrix}^T$
G_{ij}	-1.47	0	-0.732	-0.18	0	-0.224

Solve the first and the second level programming problems using FGP model to get x_1^0 and x_2^0 . Thus, the final proposed FGP model for the parametric fuzzy ML-MOFP problem is obtained as:

$$\min Z = 0.294D_{11}^- + 0.17D_{12}^- + 0.183D_{21}^- + 0.18D_{22}^- + 0.126D_{31}^- + 0.112D_{32}^-$$

subject to

$$\begin{aligned}
 & -0.512x_1 - 1.91x_2 - 4.4x_3 + D_{11}^- - D_{11}^+ = -1.47, \\
 & 0x_1 - 1.53x_2 - 1.19x_3 + D_{12}^- - D_{12}^+ = 0, \\
 & -1.897x_1 - 0.491x_2 - 1.41x_3 + D_{21}^- - D_{21}^+ = -0.732, \\
 & -0.109x_1 - 2.018x_2 + 0.071x_3 + D_{22}^- - D_{22}^+ = -0.18, \\
 & 0x_1 - 2.64x_2 - 2.138x_3 + D_{31}^- - D_{31}^+ = 0, \\
 & -0.224x_1 - 3.024x_2 + 0.336x_3 + D_{32}^- - D_{32}^+ = -0.224, \\
 & -x_1 - 2x_2 - 4x_3 + D_{11}^- \leq 0, \\
 & -x_1 - 2x_2 - x_3 + D_{12}^- \leq 0, \\
 & -2x_1 - x_2 - x_3 + D_{21}^- \leq 0, \\
 & -x_1 - 2x_2 - x_3 + D_{22}^- \leq 0, \\
 & -x_1 - 3x_2 - 2x_3 + D_{31}^- \leq 0, \\
 & -x_1 - 3x_2 + D_{32}^- \leq 0, \\
 & 2x_1 + x_2 + x_3 \leq 10, \\
 & x_1 - 2x_2 + 3x_3 \leq 8, \\
 & x_1 + 2x_2 \geq 3, \\
 & x_1 = 3, \\
 & x_2 = 0, \\
 & x_3 \geq 0, \\
 & D_{11}^-, D_{11}^+, D_{12}^-, D_{12}^+, D_{21}^-, D_{21}^+, D_{22}^-, D_{22}^+, D_{31}^-, D_{31}^+, D_{32}^-, D_{32}^+ \geq 0.
 \end{aligned}$$

Using Lingo programming, the α -compromise solution of the parametric fuzzy ML-MOFP problem is obtained at $(x_1^0, x_2^0, x_3^0) = (3, 0, 0)$.

Stage II: determination of the fuzzy stability set of the first kind $S_1(x^0, \alpha)$

To determine the fuzzy stability set of the first kind $S_1(x^0, \alpha)$ of the parametric fuzzy ML-MOFP problem, the coefficients of the linearized membership goals in the parametric form are recalculated and summarized in Table 3. and Table 4 respectively.

Table 3. The coefficients of the linearized membership goals $(c^{ij} + h^{ij}\theta)^T$ and G_{ij}

	$f_{11}(x, \theta)$	$f_{12}(x, \theta)$	$f_{21}(x, \theta)$
$(c^{ij} + h^{ij}\theta)^T$	$\begin{pmatrix} -1.1 + 0.588\theta \\ -1.91 \\ -4.11 - 0.294\theta \end{pmatrix}^T$	$\begin{pmatrix} 0 \\ -1.36 - 0.17\theta \\ -1.02 - 0.17\theta \end{pmatrix}^T$	$\begin{pmatrix} -1.714 - 0.183\theta \\ -1.04 + 0.549\theta \\ -1.41 \end{pmatrix}^T$
G_{ij}	-1.47	0	-0.732

Table 4. The coefficients of the linearized membership goals $(c^{ij} + h^{ij}\theta)^T$ and G_{ij}

	$f_{22}(x, \theta)$	$f_{31}(x, \theta)$	$f_{32}(x, \theta)$
$(c^{ij} + h^{ij}\theta)^T$	$\begin{pmatrix} -0.649 + 0.54\theta \\ -1.838 - 0.18\theta \\ 0.071 \end{pmatrix}^T$	$\begin{pmatrix} 0 \\ -2.77 + 0.126\theta \\ -2.138 \end{pmatrix}^T$	$\begin{pmatrix} -0.896 + 0.672\theta \\ -2.912 - 0.112\theta \\ 0.336 \end{pmatrix}^T$
G_{ij}	-0.18	0	-0.224

Therefore, the stability of parametric fuzzy ML-MOFP problem implies the stability of the parametric FGP model which is defined as follows:

$$\begin{aligned}
 \min Z &= 0.294D_{11}^- + 0.17D_{12}^- + 0.183D_{21}^- + 0.18D_{22}^- + 0.126D_{31}^- + 0.112D_{32}^- \\
 \text{subject to} \\
 & (-1.1 + 0.588\theta)x_1 - 1.91x_2 - (4.11 + 0.294\theta)x_3 + D_{11}^- - D_{11}^+ = -1.47,
 \end{aligned}$$

$$\begin{aligned}
 &0x_1 - (1.36 + 0.17\theta)x_2 - (1.02 + 0.17\theta)x_3 + D_{12}^- - D_{12}^+ = 0, \\
 &-(1.714 + 0.183\theta)x_1 + (-1.04 + 0.549\theta)x_2 - 1.41x_3 + D_{21}^- - D_{21}^+ = -0.732, \\
 &(-0.649 + 0.54\theta)x_1 - (1.838 + 0.18\theta)x_2 + 0.071x_3 + D_{22}^- - D_{22}^+ = -0.18, \\
 &0x_1 + (-2.77 + 0.126\theta)x_2 - 2.138x_3 + D_{31}^- - D_{31}^+ = 0, \\
 &(-0.896 + 0.672\theta)x_1 - (2.912 + 0.112\theta)x_2 + 0.336x_3 + D_{32}^- - D_{32}^+ = -0.224, \\
 &-x_1 - 2x_2 - 4x_3 + D_{11}^- \leq 0, \\
 &-x_1 - 2x_2 - x_3 + D_{12}^- \leq 0, \\
 &-2x_1 - x_2 - x_3 + D_{21}^- \leq 0, \\
 &-x_1 - 2x_2 - x_3 + D_{22}^- \leq 0, \\
 &-x_1 - 3x_2 - 2x_3 + D_{31}^- \leq 0, \\
 &-x_1 - 3x_2 + D_{32}^- \leq 0, \\
 &2x_1 + x_2 + x_3 - b_1 \leq 0, \\
 &x_1 - 2x_2 + 3x_3 - b_2 \leq 0, \\
 &x_1 + 2x_2 \geq 3, \\
 &3 + 7\alpha \leq b_1 \leq 15 - 5\alpha, \\
 &2 + 5\alpha \leq b_2 \leq 12 - 5\alpha, \\
 &x_1 = 3, \\
 &x_2 = 0, \\
 &x_3 \geq 0, \\
 &D_{11}^+, D_{11}^-, D_{12}^+, D_{12}^-, D_{21}^+, D_{21}^-, D_{22}^+, D_{22}^-, D_{31}^+, D_{31}^-, D_{32}^+, D_{32}^- \geq 0.
 \end{aligned}$$

The Lagrangean function of the above parametric FGP model follows as:

$$\begin{aligned}
 L = &0.294D_{11}^- + 0.17D_{12}^- + 0.183D_{21}^- + 0.18D_{22}^- + 0.126D_{31}^- + 0.112D_{32}^- \\
 &+ \lambda_{11}[(-1.1 + 0.588\theta)x_1 - 1.91x_2 - (4.11 + 0.294\theta)x_3 + D_{11}^- - D_{11}^+ + 1.47] \\
 &+ \lambda_{12}[0x_1 - (1.36 + 0.17\theta)x_2 - (1.02 + 0.17\theta)x_3 + D_{12}^- - D_{12}^+] \\
 &+ \lambda_{21}[(-1.714 + 0.183\theta)x_1 + (-1.04 + 0.549\theta)x_2 - 1.41x_3 + D_{21}^- - D_{21}^+ + 0.732] \\
 &+ \lambda_{22}[(-0.649 + 0.54\theta)x_1 - (1.838 + 0.18\theta)x_2 + 0.071x_3 + D_{22}^- - D_{22}^+ + 0.18] \\
 &+ \lambda_{31}[0x_1 + (-2.77 + 0.126\theta)x_2 - 2.138x_3 + D_{31}^- - D_{31}^+] \\
 &+ \lambda_{32}[(-0.896 + 0.672\theta)x_1 - (2.912 + 0.112\theta)x_2 + 0.336x_3 + D_{32}^- - D_{32}^+ + 0.224] \\
 &+ \xi_1[x_1 - 3] + \xi_2[x_2 - 0] + \mu_{11}[-x_1 - 2x_2 - 4x_3 + D_{11}^-] + \mu_{12}[-x_1 - 2x_2 - x_3 + D_{12}^-] \\
 &+ \mu_{21}[-2x_1 - x_2 - x_3 + D_{21}^-] + \mu_{22}[-x_1 - 2x_2 - x_3 + D_{22}^-] \\
 &+ \mu_{31}[-x_1 - 3x_2 - 2x_3 + D_{31}^-] + \mu_{32}[-x_1 - 3x_2 + D_{32}^-] + v_1[2x_1 + x_2 + x_3 - b_1] \\
 &+ v_2[x_1 - 2x_2 + 3x_3 - b_2] + v_3[-x_1 - 2x_2 + 3] + \eta_1[b_1 - 15 + 5\alpha] + \eta_2[b_2 - 12 + 5\alpha] \\
 &+ \phi_1[-b_1 + 3 + 7\alpha] + \phi_2[-b_2 + 2 + 5\alpha] + \psi[-x_3] + \gamma_{11}[-D_{11}^-] + \delta_{11}[-D_{11}^+] \\
 &+ \gamma_{12}[-D_{12}^-] + \delta_{12}[-D_{12}^+] + \gamma_{21}[-D_{21}^-] + \delta_{21}[-D_{21}^+] + \gamma_{22}[-D_{22}^-] + \delta_{22}[-D_{22}^+] \\
 &+ \gamma_{31}[-D_{31}^-] + \delta_{31}[-D_{31}^+] + \gamma_{32}[-D_{32}^-] + \delta_{32}[-D_{32}^+].
 \end{aligned}$$

where, $\theta, \lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}, \lambda_{31}, \lambda_{32}, \xi_1, \xi_2 \in R$, and $\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \mu_{31}, \mu_{32}, v_1, v_2, v_3, \phi_1, \phi_2, \eta_1, \eta_2, \psi \geq 0$ also $\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}, \gamma_{31}, \gamma_{32}, \delta_{11}, \delta_{12}, \delta_{21}, \delta_{22}, \delta_{31}, \delta_{32} \geq 0$ are the Lagrange multipliers. Therefore, the Kuhn-Tucker necessary optimality conditions corresponding to the parametric FGP model follows as:

$$\begin{aligned}
 \frac{\partial L}{\partial x_1} = &(-1.1 + 0.588\theta)\lambda_{11} - (1.714 + 0.183\theta)\lambda_{21} + (-0.649 + 0.54\theta)\lambda_{22} \\
 &+ (-0.896 + 0.672\theta)\lambda_{32} + \xi_1 - \mu_{11} - \mu_{12} - 2\mu_{21} - \mu_{22} - \mu_{31} - \mu_{32} + 2v_1 \\
 &+ v_2 - v_3 = 0,
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial L}{\partial x_2} = &-1.91\lambda_{11} - (1.36 + 0.17\theta)\lambda_{12} + (-1.04 + 0.549\theta)\lambda_{21} - (1.838 + 0.18\theta)\lambda_{22} \\
 &+ (-2.77 + 0.126\theta)\lambda_{31} - (2.912 + 0.112\theta)\lambda_{32} + \xi_2 - 2\mu_{11} - 2\mu_{12} - \mu_{21} - 2\mu_{22} \\
 &- 3\mu_{31} - 3\mu_{32} + v_1 - 2v_2 - 2v_3 = 0,
 \end{aligned}$$

$$\frac{\partial L}{\partial x_3} = -(4.11 + 0.294\theta)\lambda_{11} - (1.02 + 0.17\theta)\lambda_{12} - 1.41\lambda_{21} + 0.071\lambda_{22} - 2.138\lambda_{31} \\ + 0.336\lambda_{32} - 4\mu_{11} - \mu_{12} - \mu_{21} - \mu_{22} - 2\mu_{31} + v_1 + 3v_2 - \psi = 0,$$

$$\frac{\partial L}{\partial b_1} = -v_1 + \eta_1 - \phi_1 = 0,$$

$$\frac{\partial L}{\partial b_2} = -v_2 + \eta_2 - \phi_2 = 0,$$

$$\frac{\partial L}{\partial D_{11}^-} = 0.294 + \lambda_{11} + \mu_{11} - \gamma_{11} = 0,$$

$$\frac{\partial L}{\partial D_{11}^+} = -\lambda_{11} - \delta_{11} = 0,$$

$$\frac{\partial L}{\partial D_{12}^-} = 0.17 + \lambda_{12} + \mu_{12} - \gamma_{12} = 0,$$

$$\frac{\partial L}{\partial D_{12}^+} = -\lambda_{12} - \delta_{12} = 0,$$

$$\frac{\partial L}{\partial D_{21}^-} = 0.183 + \lambda_{21} + \mu_{21} - \gamma_{21} = 0,$$

$$\frac{\partial L}{\partial D_{21}^+} = -\lambda_{21} - \delta_{21} = 0,$$

$$\frac{\partial L}{\partial D_{22}^-} = 0.18 + \lambda_{22} + \mu_{22} - \gamma_{22} = 0,$$

$$\frac{\partial L}{\partial D_{22}^+} = -\lambda_{22} - \delta_{22} = 0,$$

$$\frac{\partial L}{\partial D_{31}^-} = 0.126 + \lambda_{31} + \mu_{31} - \gamma_{31} = 0,$$

$$\frac{\partial L}{\partial D_{31}^+} = -\lambda_{31} - \delta_{31} = 0,$$

$$\frac{\partial L}{\partial D_{32}^-} = 0.112 + \lambda_{32} + \mu_{32} - \gamma_{32} = 0,$$

$$\frac{\partial L}{\partial D_{32}^+} = -\lambda_{32} - \delta_{32} = 0,$$

$$\mu_{11}[-x_1 - 2x_2 - 4x_3 + D_{11}^-] = 0, \quad i. e. \mu_{11} = 0,$$

$$\mu_{12}[-x_1 - 2x_2 - x_3 + D_{12}^-] = 0, \quad i. e. \mu_{12} = 0,$$

$$\mu_{21}[-2x_1 - x_2 - x_3 + D_{21}^-] = 0, \quad i. e. \mu_{21} = 0,$$

$$\mu_{22}[-x_1 - 2x_2 - x_3 + D_{22}^-] = 0, \quad i. e. \mu_{22} = 0,$$

$$\mu_{31}[-x_1 - 3x_2 - 2x_3 + D_{31}^-] = 0, \quad i. e. \mu_{31} = 0,$$

$$\mu_{32}[-x_1 - 3x_2 + D_{32}^-] = 0, \quad i. e. \mu_{32} = 0,$$

$$v_1[2x_1 + x_2 + x_3 - b_1] = 0, \quad i. e. v_1 = 0,$$

$$v_2[x_1 - 2x_2 + 3x_3 - b_2] = 0, \quad i. e. v_2 = 0,$$

$$v_3[-x_1 - 2x_2 + 3] = 0, \quad i. e. v_3 \neq 0,$$

$$\eta_1[b_1 - 15 + 5\alpha] = 0, \quad i. e. \eta_1 = 0,$$

$$\eta_2[[b_2 - 12 + 5\alpha]] = 0, \quad i. e. \eta_2 = 0,$$

$$\phi_1[-b_1 + 3 + 7\alpha] = 0, \quad i. e. \phi_1 = 0,$$

$$\phi_2[-b_2 + 2 + 5\alpha] = 0, \quad i. e. \phi_2 = 0,$$

$$\psi[-x_3] = 0, \quad i. e. \psi \neq 0,$$

$$\gamma_{11}[-D_{11}^-] = 0, \quad i. e. \gamma_{11} = 0,$$

$$\delta_{11}[-D_{11}^+] = 0, \quad i. e. \delta_{11} \neq 0,$$

$$\gamma_{12}[-D_{12}^-] = 0, \quad i. e. \gamma_{12} \neq 0,$$

$$\delta_{12}[-D_{12}^+] = 0, \quad i. e. \delta_{12} \neq 0,$$

$$\gamma_{21}[-D_{21}^-] = 0, \quad i. e. \gamma_{21} = 0,$$

$$\begin{aligned}
 \delta_{21}[-D_{21}^+] &= 0, & i.e. \delta_{21} &\neq 0, \\
 \gamma_{22}[-D_{22}^-] &= 0, & i.e. \gamma_{22} &= 0, \\
 \delta_{22}[-D_{22}^+] &= 0, & i.e. \delta_{22} &\neq 0, \\
 \gamma_{31}[-D_{31}^-] &= 0, & i.e. \gamma_{31} &\neq 0, \\
 \delta_{31}[-D_{31}^+] &= 0, & i.e. \delta_{31} &\neq 0, \\
 \gamma_{32}[-D_{32}^-] &= 0, & i.e. \gamma_{32} &= 0, \\
 \delta_{32}[-D_{32}^+] &= 0, & i.e. \delta_{32} &\neq 0, \\
 -x_1 - 2x_2 - 4x_3 + D_{11}^- &\leq 0, \\
 -x_1 - 2x_2 - x_3 + D_{12}^- &\leq 0, \\
 -2x_1 - x_2 - x_3 + D_{21}^- &\leq 0, \\
 -x_1 - 2x_2 - x_3 + D_{22}^- &\leq 0, \\
 -x_1 - 3x_2 - 2x_3 + D_{31}^- &\leq 0, \\
 -x_1 - 3x_2 + D_{32}^- &\leq 0, \\
 2x_1 + x_2 + x_3 - b_1 &\leq 0, \\
 x_1 - 2x_2 + 3x_3 - b_2 &\leq 0, \\
 -x_1 - 2x_2 + 3 &\leq 0, \\
 3 + 7\alpha \leq b_1 \leq 15 - 5\alpha, \\
 2 + 5\alpha \leq b_2 \leq 12 - 5\alpha, \\
 x_1 &= 3, \\
 x_2 &= 0, \\
 x_3 &\geq 0, \\
 D_{11}^-, D_{11}^+, D_{12}^-, D_{12}^+, D_{21}^-, D_{21}^+, D_{22}^-, D_{22}^+, D_{31}^-, D_{31}^+, D_{32}^-, D_{32}^+ &\geq 0.
 \end{aligned}$$

Solving the above system of equations we get: $\lambda_{11} = -0.294, \lambda_{21} = -0.183, \lambda_{22} = -0.18, \lambda_{32} = -0.112, -0.17 \leq \lambda_{12} \leq 0, -0.126 \leq \lambda_{31} \leq 0, \delta_{11} = 0.294, \delta_{12} = -\lambda_{12}, \delta_{21} = 0.183, \delta_{22} = 0.18, \delta_{31} = -\lambda_{31}, \delta_{32} = 0.112$ and $\xi_1, \xi_2 \in R$ also, $\mu_{11} = \mu_{12} = \mu_{21} = \mu_{22} = \mu_{31} = \mu_{32} = \nu_1 = \nu_2 = \eta_1 = \eta_2 = \phi_1 = \phi_2 = \gamma_{11} = \gamma_{21} = \gamma_{22} = \gamma_{32} = 0$, and $\gamma_{12}, \gamma_{31}, \psi, v_3 \geq 0$.

The above system of equations is reduced to the following system of equations:

$$\begin{aligned}
 &(-1.1 + 0.588\theta)\lambda_{11} - (1.714 + 0.183\theta)\lambda_{21} + (-0.649 + 0.54\theta)\lambda_{22} + (-0.896 + 0.672\theta) \\
 &\lambda_{32} + \xi_1 - v_3 = 0, \\
 &-1.91\lambda_{11} - (1.36 + 0.17\theta)\lambda_{12} + (-1.04 + 0.549\theta)\lambda_{21} - (1.838 + 0.18\theta)\lambda_{22} + (-2.77 + \\
 &0.126\theta)\lambda_{31} - (2.912 + 0.112\theta)\lambda_{32} + \xi_2 - 2v_3 = 0, \\
 &(-4.11 - 0.294\theta)\lambda_{11} - (1.02 + 0.17\theta)\lambda_{12} - 1.41\lambda_{21} + 0.071\lambda_{22} - 2.138\lambda_{31} + 0.336\lambda_{32} \\
 &- \psi = 0,
 \end{aligned}$$

Therefore, the fuzzy stability set of the first kind for the parametric fuzzy ML-MOFP problem of the numerical example is given by:

$$S_1(3,0,0,\alpha) = \left\{ \begin{array}{l} \theta \in R, \\ \alpha \in [0,1] \end{array} \left\{ \begin{array}{l} 3.679 + [-0.281 - 0.34\lambda_{12} + 0.126\lambda_{31}]\theta - 2.38\lambda_{12} - 4.91\lambda_{31} \\ + \xi_1 + \xi_2 - 3v_3 - \psi = 0, \quad \xi_1, \xi_2 \in R, v_3, \psi \geq 0, \\ -0.17 \leq \lambda_{12} \leq 0, \quad -0.126 \leq \lambda_{31} \leq 0, \\ 3 + 7\alpha \leq b_1 \leq 15 - 5\alpha, \quad 2 + 5\alpha \leq b_2 \leq 12 - 5\alpha. \end{array} \right. \right\}$$

7 Conclusion and Summary

In the present research, the fuzzy stability set of the first kind for the parametric fuzzy ML-MOFP problem has been presented. Some basic stability notions like the set of feasible parameters and the solvability set have been defined for such problem. Moreover, FGP approach has been extended to find an α -Pareto optimal solution for parametric fuzzy ML-MOFP problem. In FGP approach, the membership functions for

the defined fuzzy goals are developed. Also, in the proposed approach, linearization of membership goals of the objective functions is presented. Then, the highest degree of each of these membership goals is achieved by minimizing the sum of the negative deviational variables. After obtaining the compromise solution, the Lagrangian function is formulated. To obtain the fuzzy stability set of the first kind, the Kuhn-Tucker necessary optimality conditions are developed. A procedure has been suggested for the determination of the fuzzy stability set of the first kind for such problem.

Several open points for research in the area of parametric ML-MOFP problems, from our point of view, to be studied in the future. Some of these points are given in the following:

1. Interactive algorithm is needed for dealing with parametric fuzzy multi-level multi-objective fractional programming with fuzzy demands.
2. Interactive algorithm is needed for dealing with parametric rough multi-level multi-objective fractional programming.
3. Fuzzy goal programming algorithm is required for treating parametric multi-level multi-objective fractional in rough environment.

Competing Interests

Authors have declared that no competing interests exist.

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