Modeling of Nigeria's Economic Growth Rate: A Probability Distribution Fitting Approach

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Authors' contributions

This work was carried out in collaboration between all authors. Author ECN designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author CNE managed the analyses of the study and the literature searches. All authors read and approved the final manuscript.

Original Research Article

Abstract

This study examined the probability distribution that best described the quarterly economic growth rate of Nigeria between 1960- 2015. The study collected secondary data from Central Bank of Nigeria (CBN) Statistical Bulletin 2015 on Gross Domestic Product to compute the economic growth rate of Nigeria. Six theoretical statistical distributions were fitted via Normal Distribution, Logistic Distribution, Laplace Distribution, Cauchy Distribution, Gumbel (Largest Extreme Value) Distribution and Generalized Logistic Distribution. The Laplace Distribution fitted the data as confirmed by Kolmogorov Simonov goodness of fit test, Akaike Information Criteria and Bayes Information Criteria. The probabilities of economic growth rate behaviours were obtained from the best fit distribution. The analysis showed that the chance of obtaining a negative quarterly economic growth rate is 28%. The chance of an economic recession is 8%. Also, the probability of having a positive single digit quarterly economic growth rate is 46%. In addition, having a double digit positive quarterly economic growth rate is 26%.

Keywords: Economic growth rate; probability distribution; statistical modeling; goodness-of-fit test.

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1 Introduction

The Gross Domestic Product (GDP) of a country at the market price is defined as final goods and services produced within a country in a given period of time. The GDP has always been used as an indicator to measure economic performance and growth in many countries. As a measure of economic growth, a rising gross domestic product indicates that an economy is expanding in terms of production of goods and services, while a decreasing gross domestic product means that an economy is contracting. This means that a gross domestic product of a country exhibits irregular behaviour in both short run and long run. Therefore, how fast an economy is rising or contracting becomes important to measure it. This measure is called the economic growth rate of an economy. The quarterly increase in gross domestic product percentage shows the economic growth rate. The volatility of this economic growth rate is important to economic and public policy makers and as such the need to understand its behaviour.

In Nigeria, the quarterly economic growth rate from 1960 to 2015 has exhibited enormous variability. This can be seen in the graph (Fig. 1.) of the time sequence plot for 244 observation points of economic growth rate (EGR %) in percentage.

The high variability of economic growth rate in Nigeria affects policy thrusts, thereby making economic planning difficult for economic managers. In this regard, to accurately understand the economic growth rate gyration in Nigeria, a probability density distribution is required to model the pattern of movement of this primary economic indicator. Consequently, the scatter plot of the economic growth rate (EGR %) within the stated period will assist to identify candidate probability distributions that can be used to model the economic growth rate pattern.

The scatter plot of the economic growth rate (EGR %) for the 244 observation points in Fig. 2.0 shows a distribution with majority of the observations clustering around zero. Hence, probability distributions with this shape were identified and used for modeling Nigeria's economic growth rate. Therefore, this research work on Modeling of Nigeria's Economic Growth Rate: A Probability Distribution Fitting Approach will mainly fit theoretical probability distributions (Normal, Laplace, Logistic, Gumbel (large extreme value), Cauchy, Generalized Logistic) in order to find which probability distribution that can best describe the quarterly economic growth rate of Nigeria and make probability forecasts. This study covered the period, 1960 to 2015.
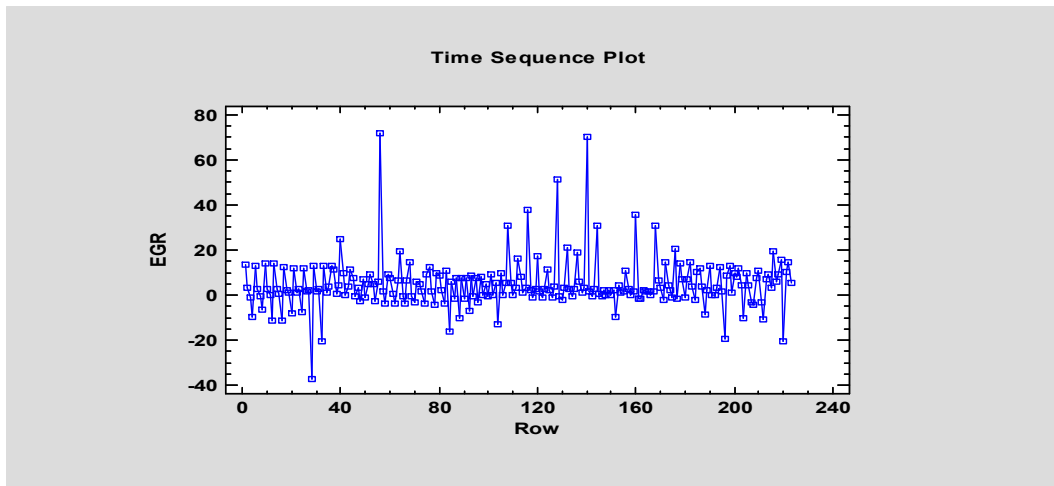

Fig. 1. The time sequence plot of quarterly economic growth rates (EGR %) of Nigeria from 1960 to 2015 for 244 observation points
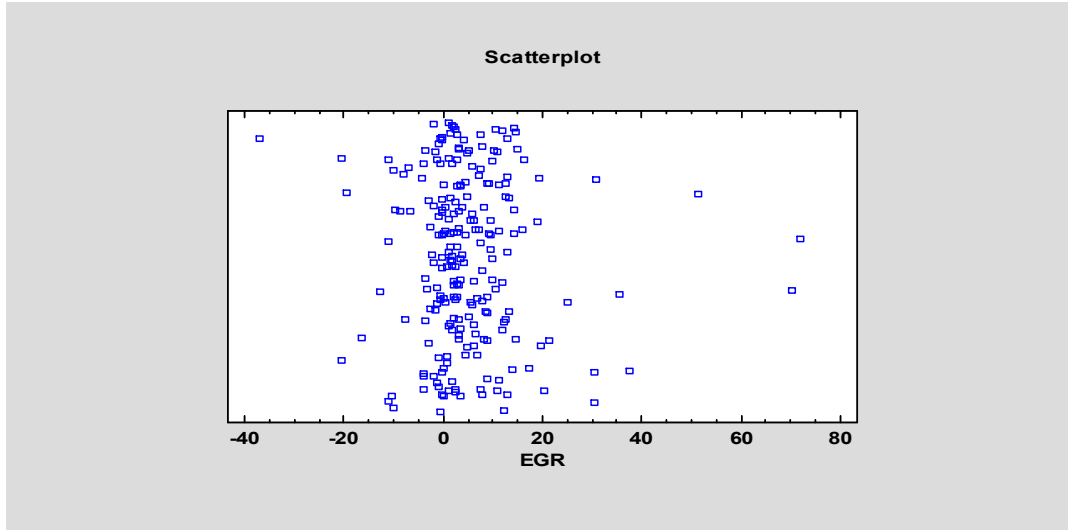*Source: Author's computation and Statgraphics output*

Fig. 2. The Scatter plot of quarterly economic growth rates (EGR %) of Nigeria from 1960 to 2015 for 244 observation points
Source: Author's computation and Statgraphics output

2 Literature Review

The empirical review of this study is to discover what has been done in the past regarding this area of research. Few studies have been done to investigate the probability distribution of the economic growth rate in developed and developing countries.

In their work, "How do Output Growth Rate Distribution Look Like? Some Cross-Country Times Series Evidence", Fagiolo, Napoletano and Roventini [1] investigated the statistical properties of the within-country gross domestic product (GDP) and industrial production (IP) growth-rate distributions. They opined that many empirical contributions have recently pointed out that cross-section growth rates of firms, industries and countries all follow Laplace distributions. In their work, they tested whether also within-country; time-series GDP and IP growth rates can be approximated by tent-shaped distributions. They fitted output growth rates with the exponential-power (Subbotin) family of densities, which includes as particular cases both Gaussian and Laplace distributions. They found that, for a large number of OECD (Organization for Economic Cooperation and Development) countries including the US, both GDP and IP growth rates are Laplace distributed. Moreover, they showed that fat-tailed distributions robustly emerge even after controlling for outliers, autocorrelation and heteroscedasticity.

Also, Stockhammer and Oller [2] examined "the Probability Distribution of Economic Growth". They said normality is often mechanically and without sufficient reason assumed in econometric models. In their paper three important and significantly heteroscedastic GDP series were studied. Heteroscedasticity was removed and the distributions of the filtered series were then compared to Normal, Normal Mixture and Normal - Asymmetric Laplace (NAL) distributions. NAL represents a skewed and leptokurtic distribution, which is in line with the Aghion and Howitt [3] model for economic growth, based on Schumpeter's idea of creative destruction. Statistical properties of the NAL distributions were provided and it is shown that NAL competes well with the alternatives.

In another study, "Fitting Probability Distribution to Economic Growth: A Maximum Likelihood Approach", Hassan and Stockhammer [4] pronounced that the growth rate of the gross domestic product (GDP) usually carries heteroscedasticity, asymmetry and fat-tails. In that study three important and significantly heteroscedastic GDP series were examined. A Normal, normal-mixture, normal-asymmetric Laplace

distribution and a Student's t-Asymmetric Laplace (TAL) distribution mixture were considered for distributional fit comparison of GDP growth series after removing heteroscedasticity. The parameters of the distributions have been estimated using the maximum likelihood method. Based on the results of different accuracy measures, goodness-of-fit tests and plots, they found out that in the case of asymmetric, heteroscedastic and highly leptokurtic data the TAL-distribution fits better than the alternatives. In the case of asymmetric, heteroscedastic but less leptokurtic data the NM fit is superior.

More so, Ascari, Fagiolo and Roventini [5] investigated flat-tail distributions and business cycle models. The findings showed that macroeconomic variables are seldom normally distributed. From their study, the distributions of aggregate output growth-rate time series of many OECD countries were well approximated by symmetric exponential-power (EP) densities, with Laplace fat tails. In that work, they assessed whether Real Business Cycle (RBC) and standard medium-scale New-Keynesian (NK) models were able to replicate this statistical regularity. They simulated both models drawing Gaussian- vs Laplace-distributed shocks and they explored the statistical properties of simulated time series. Their results casted doubts on whether RBC and NK models are able to provide a satisfactory representation of the transmission mechanisms linking exogenous shocks to macroeconomic dynamics.

In addition, Podobnik, Horvatic, Njavro and Stanley [6] studied scaling of growth rate volatility for six macroeconomic variables: public debt, public health expenditures, exports of goods, government consumption expenditures, total exports of goods and services, and total imports of goods and services. For each variable, they found (i) that the distribution of the growth rate residuals approximately follows a double exponential (Laplace) distribution and (ii) that the standard deviation of growth rate residuals scales according to the size of the variable as a power law, with a scaling exponent similar to the scaling exponent found for GDP [Economics Letters 60, 335 [7]]. They hypothesized that the volatility scaling they discovered for these GDP constituents caused the volatility scaling found in GDP data.

Nevertheless, Franke [8] examined how fat-tailed output growth in United States of America is. To this end, he focused on the shape parameter b of the exponential power distribution (EPD), the two polar values of which constitute the normal distribution and the Laplace distribution with its fatter tails, respectively. His paper first warned against premature conclusions that neglect a structural break in output volatility. Distinguishing the two periods of the Great Inflation and the Great Moderation, it is then found that for quarterly industrial production (IP) the Laplacian cannot be rejected in both periods. By contrast, for monthly IP and quarterly GDP or firm output, the evidence is mixed and even contradictory. His question on whether non-normality can be considered a new stylized fact for macroeconomic modelling has, therefore, no unambiguous answer.

In all these studies, the researchers focused mainly on using various forms of Laplace distributions to model economic growth rate. They all agreed that Laplace Distribution is a good probability model for economic growth rate. However, all the studies failed to use other forms of probability distributions other than Laplace distribution variants and more so did not forecast probabilities for different types of economic growth rate behaviours. Therefore, the focus of this research work is to fill this part of the area of study in relation to the modelling of Nigeria's economic growth rate using probability distribution, thereby closing this yawning gap in knowledge.

3 Methodology

The method of analysis adopted in this study is the method of goodness-of-fit test. Since interest is to determine the distribution that best describes the pattern in the economic growth rate of Nigeria. Also, the parameters of the distributions were estimated using the maximum likelihood estimation. In addition, the data was used to compute various descriptive statistics such as mean median, variance, skewness and kurtosis. Descriptive statistics from the fitted probability distributions were computed to check if there were similarities and discrepancies. The nature of the data shows that it is of the continuous type. Therefore, this suggested six types of theoretical probability distributions that could fit the data, namely, Normal Distribution, Laplace Distribution, Logistic Distribution, Cauchy Distribution, Gumbel (Largest extreme

value) Distribution and Generalized Logistic Value Distribution. Consequently, the data were assessed for various statistical tests such as, test of stationarity, a test of heteroscedasticity and test of autocorrelation. Subsequently, two model selection criteria were used to select the probability distribution model that best fits the economic growth data of Nigeria, these are Akaike Information Critera (AIC) and Bayes Information Criteria (BIC). The method of goodness-of-fit used in this study is the Kolmogorov-Smirnov test. Consequently, the following hypotheses were tested in this study. Hence, the decision criteria used to report the outcome of statistical hypothesis in this work is the p-Value. Thus, acceptance or rejection of a hypothesis will be determined by comparing the p-Value to the chosen level of significance α. P-Value Rule: Reject the null hypothesis when the p-Value is less than, or equal to, the level of significance α. That is, if p $\leq$ α reject H_0, if p > α then do not reject H_0 .

3.1 Theoretical probability distributions

The theoretical probability distributions (Normal Distribution, Laplace Distribution, Logistic Distribution, Cauchy Distribution, Gumbel (Largest extreme value) Distribution and Generalized Logistic Distribution were used to model the economic growth rate.

3.2 Normal distribution

The normal distribution is applicable to a very wide range of phenomena and is the most widely used distribution in statistics. The probability density function of the normal distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \qquad -\infty < x < \infty, \sigma > 0 \tag{1}$$

Where μ is the location parameter and σ is the scale parameter. [9]

3.3 Laplace distribution

The Laplace distribution also known as double exponential distribution is heavy tailed distribution more than the normal distribution. The probability density function of the Laplace distribution is:

$$f(x) = \frac{1}{2\beta}\exp\left(-\left|\frac{x-\mu}{\beta}\right|\right) \qquad -\infty < x < \infty \tag{2}$$

Where the location parameter −∞ < μ < ∞ is the mean. Scale parameter β > 0. [10]

3.4 Logistic distribution

The logistic distribution is most times used for growth model and it has similar shape of the normal distribution. The probability density function of the logistic distribution is:

$$f(x) = \frac{e^{-\frac{x-\mu}{\sigma}}}{\sigma\left(1+e^{-\frac{x-\mu}{\sigma}}\right)^2}; \qquad -\infty < \mu < \infty, \mu > 0, \sigma > 0$$

$$\tag{3}$$

Where the location parameter −∞ < μ < ∞ is the mean. Scale parameter σ > 0. [11]

3.5 Cauchy distribution

The Cauchy distribution is unimodal and symmetric, with much heavier tails than the normal distribution. The probability density function is symmetric about α, with upper and lower quartiles, $\alpha \pm \beta$. The Cauchy distribution is of mathematical interest due to the absence of defining moments. The probability density function of the Cauchy distribution is:

$$f(x, \alpha, \beta) = \frac{\beta}{\pi\left(\beta^2 + (x - \alpha)^2\right)} \quad -\infty < x < \infty, \beta > 0 \tag{4}$$

Where α is the location parameter and β the scale parameter. [12]

3.6 Gumbel (largest extreme value) distribution

The Gumbel (largest extreme value) distribution is one of the Gumbel distributions. The distribution is skewed to right and can be used to model logarithm of times, strength and life of a product. The probability density function of the Gumbel (largest extreme value) distribution is:

$$f(x) = \frac{1}{\beta} - e^{\frac{x-\mu}{\beta}} e^{-e^{\frac{x-\mu}{\beta}}} \quad -\infty < x < \infty \tag{5}$$

Where μ is the location parameter and β is the scale parameter. The distribution has the following characteristics: [13]

3.7 Generalised logistic distribution

The Generalized Logistic Distribution is a logistic parameter distribution that can be used to model the growth of a natural phenomenon. The probability density function (p.d.f) is

$$f(x) = \frac{1}{B(\alpha, \beta)} \frac{\lambda e^{-\lambda \beta x}}{\left(1 + e^{-\lambda x}\right)^{\alpha + \beta}}, -\infty < x < \infty, \alpha, \beta > 0 \tag{6}$$

Where $B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}$ and λ is the scale parameter. Also α is the location parameter, and β is the shape parameter. [13]

4. Analysis of Data

4.1 Trend movement of quarterly economic growth rate in Nigeria

The trend movement of quarterly economic growth rate in Fig. 4.0 shows that between 1960 to 1966 when Nigeria started operating as an independent nation, the quarterly economic growth rate showed a mild volatility with the highest positive quarterly economic growth rate to be 14.10% and the worst negative quarterly economic growth rate to be -11.19%. Also, with the discovery of crude oil in Nigeria, as a result the country witnessed an upsurge in economic activities between the years 1970 to 1980 thereby leading to high volatility in her quarterly economic growth rate. The highest positive quarterly economic growth rate with that period was 71.99% and the worst negative quarterly economic growth rate then was -3.86%. The adoption structural Adjustment Programme (SAP) in 1986 affected the economic activities of the country for

over a decade. The period between the years 1986 to 1999 showed a strong degeneration in country's economic activities. Nevertheless, the introduction of democratic government in the year 1999 till 2015 brought an increase in economic activities of the country with its associated variability.

4.2 Descriptive analysis of the variable

In considering the quarterly periods of the years 1960 to 2015 used for this research work, table 4.1 shows that the average quarterly economic growth rate was 4.75%. The median value for the quarterly economic growth is found be 2.88% while, standard deviation from the mean for was 11.30%. The distribution of the variable showed it is positively skewed to the right since its value is positively different from zero. From Table 4.1 the skewness value of the quarterly economic growth rate is 2.09. The kurtosis value of the quarterly economic growth rate was 11.88 and greater than 3.0; this means that the curve of the variable is platykurtic in nature.
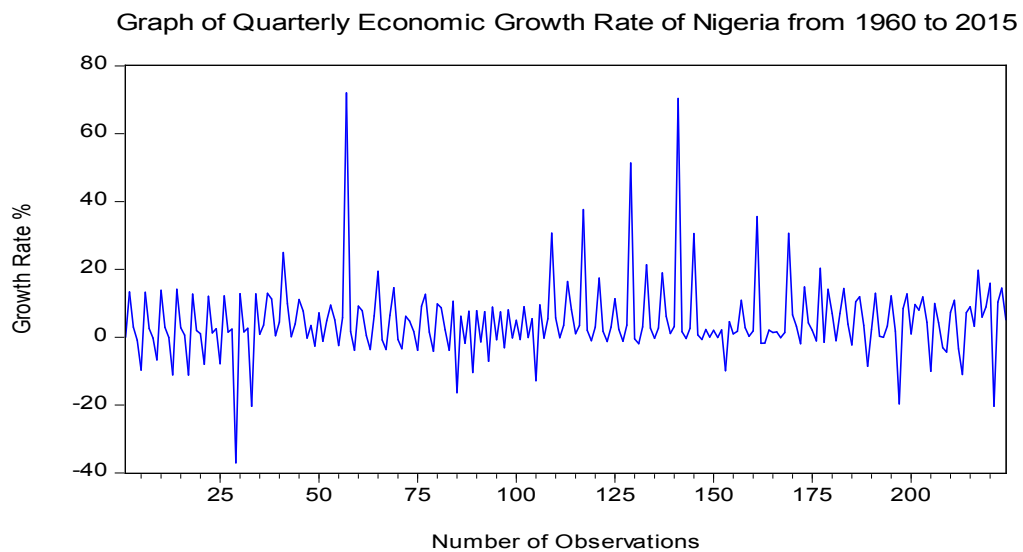

Fig. 4. Trend movement of quartely economic growth rate from 1960 To 2015
Source: Author's computation and Eviews 7.1 Output

Table 4.1. The descriptive statistics of the quarterly economic growth rate (EGR)

Mean	Median	Standard deviation	Skewness	Kurtosis
4.75%	2.88%	11.30%	2.09	11.88

Source: Author's computation and Eviews 7.1 Output

4.3 Statistical tests of the variable

4.3.1 Unit root test

The quarterly economic growth rate (EGR) was verified for stationarity by subjecting it to a unit root test. In order to determine if the series of economic growth rate is stationary, an AR (2) process of the variables is formed as:

$$EGR_t = \rho_1 EGR_{t-1} + \rho_2 EGR_{t-2} + \varepsilon_t$$

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$$EGR_t = 0.130920\,EGR_{t-1} + 0.190842\,EGR_{t-2} + \varepsilon_t$$

Therefore, the characteristic equation becomes

$$1 - 0.130920Z - 0.190842Z^2 = 0$$

Solving this quadratic characteristics equation to obtain its characteristics roots, we obtain

$R_1 = -2.6576530964045$
$R_2 = 1.9716405834351$

Therefore, the roots lie outside the unit circle and they are real numbers, hence we conclude that EGR_t is stationary since the absolute value of the roots is greater one.

4.3.2 Jarque-Bera (Jb) test of normality

Table 4.3 shows the normality test of the quarterly economic growth rate used for this research study. This is to ascertain if the values of a variable are from a normally distributed population. From the table, the quarterly economic growth rate is not from a normally distributed population since its p-value of 0.0000 is less than the hypothesised significant value of 0.05.

4.3.3 White heteroscedasticity test

Table 4.4 shows the white heteroscedasticity test of the quarterly economic growth rate. This is to ascertain if the error obtained from the autoregressive model of order two (2) while testing for stationarity has a constant variance. From the table, the quarterly economic growth rate has a constant error variance since its Prob.-F-statistic of 0.6560 is greater than the Prob.- Chi-Square of 0.6495. This indicates that there is no heteroscedasticity problem in the variable.

4.3.4 Breusch-godfrey serial correlation LM test

Table 4.5 shows the Breusch-Godfrey serial correlation test of the quarterly economic growth rate. This is to determine if there is any serial correlation between the values of the variable. From the table, the values of quarterly economic growth rate are not serially correlated since Prob.-F-statistic of 0.2803 is greater than the Prob. - Chi-Square of 0.2761. This shows that the values of the quarterly economic growth rate are independent of one another.

Table 4.2. Estimated AR (2) coefficients

Dependent variable: EGR				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.261421	0.880611	7.110316	0.0000
EGR(-1)	0.130920	0.066258	-1.975908	0.0494
EGR(-2)	0.190842	0.066343	-2.876605	0.0044
R-squared	0.048032	Mean dependent var		4.737650
Adjusted R-squared	0.039338	S.D. dependent var		11.33291
S.E. of regression	11.10776	Akaike info criterion		7.666587
Sum squared resid	27020.74	Schwarz criterion		7.712569
Log likelihood	-847.9911	Hannan-Quinn criter.		7.685151
F-statistic	5.524901	Durbin-Watson stat		2.028341
Prob(F-statistic)	0.004562			

Source: Author's computation and Eviews 7.1 Output

4.4 Goodness of fit test

4.4.1 Normal distribution

Hypothesis I: H_{01} **:** The quarterly economic growth rate of Nigeria follows the theoretical Normal distribution.

Fig. 4.1 shows the plot of the normal distribution on the histogram of quarterly economic growth of Nigeria. From the graph, it can be deduced that normal distribution does not fit the data of the variable. Also, from Table 4.6, the p-value of 0.0007 from the Kolmogorov-Smirnov test indicates that it is less than 0.05 significant levels. Therefore we conclude that normal distribution is not a good fit for the quarterly economic growth rate data in Nigeria.

Table 4.3. The Jarque-Bera test of normality computation for quarterly economic growth rate (EGR)

Variables: EGR		
Jarque-Bera	Observations	Probability
1415.308	224	0.0000

Source: Author's computation and Eviews 7.1 Output

Table 4.4. The white heteroscedasticity test for quarterly economic growth rate (EGR)

Heteroscedasticity test: White			
F-statistic	0.657603	Prob. F(5,216)	0.6560
Obs*R-squared	3.328679	Prob. Chi-Square(5)	0.6495
Scaled explained SS	22.80032	Prob. Chi-Square(5)	0.0004

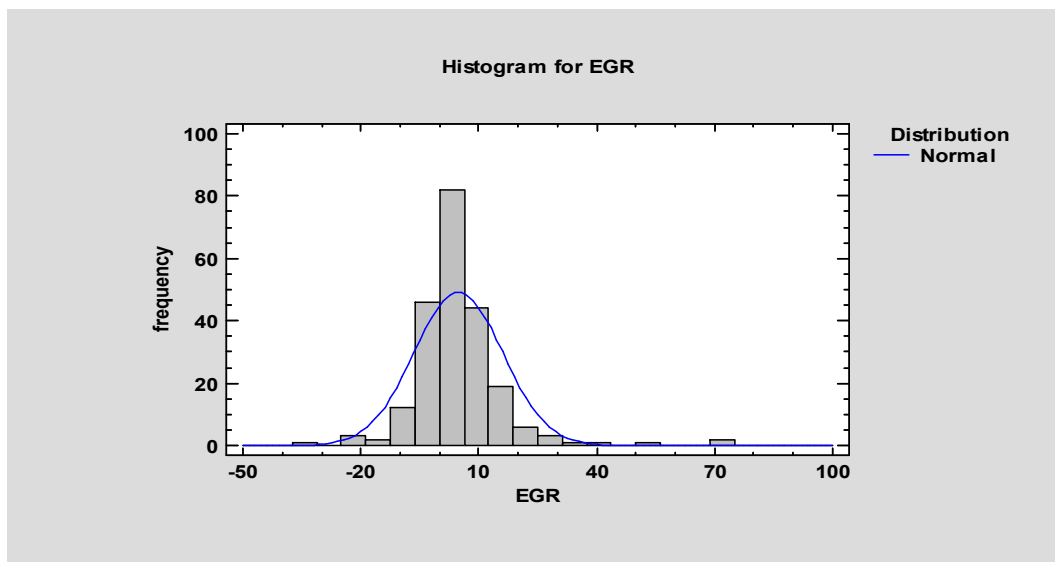*Source: Author's computation and Eviews 7.1 Output*

Fig. 4.1. Plot of Normal distribution on the histogram of quarterly economic growth rate
Source: Author's computation and Eviews 7.1 Output

Table 4.5. The Breusch-Godfrey serial correlation test for quarterly economic growth rate (EGR)

Breusch-Godfrey serial correlation LM test:			
F-statistic	1.171302	Prob. F(1,218)	0.2803
Obs*R-squared	1.186419	Prob. Chi-Square(1)	0.2761

Source: Author's computation and Eviews 7.1 Output

Table 4.6. Goodness-of-fit test for EGR using normal distribution

Normal distribution				
Mean	**Standard deviation**	**Log likelihood**	**Kolmogorov-Smirnov D**	**P-Value**
4.78%	11.32%	-857.082	0.13332	0.0007

Source: Author's computation and Eviews 7.1 Output

4.4.2 Laplace distribution

Hypothesis II

H_{02}: The quarterly economic growth rate of Nigeria follows the theoretical Laplace distribution.

Fig. 4.2 shows the plot of Laplace distribution on the histogram of quarterly economic growth of Nigeria. The graph indicates that Laplace distribution mirrored the histogram of the variable. Also, from Table 4.7, the p-value of 0.0581 from the Kolmogorov-Smirnov test shows that it is greater than 0.05 significant levels. Therefore we conclude that Laplace distribution is a good fit for the quarterly economic growth rate data in Nigeria.
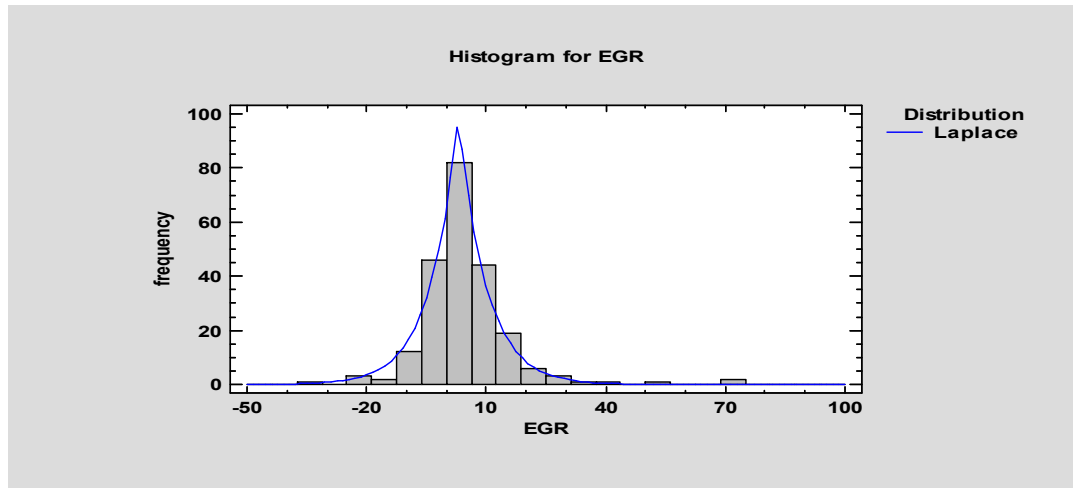

Fig. 4.2. Plot of Laplace distribution on the histogram of quarterly economic growth rate
Source: Author's computation and Eviews 7.1 Output

Table 4.7. Goodness-of-fit test for EGR using Laplace distribution

Laplace distribution				
Mean	**Standard deviation**	**Log likelihood**	**Kolmogorov-Smirnov D**	**P-Value**
4.78%	8.01%	-807.50	0.102733	0.0581

4.4.3 Logistic distribution

Hypothesis III

H_{03}**:** The quarterly economic growth rate of Nigeria follows the theoretical Logistic distribution
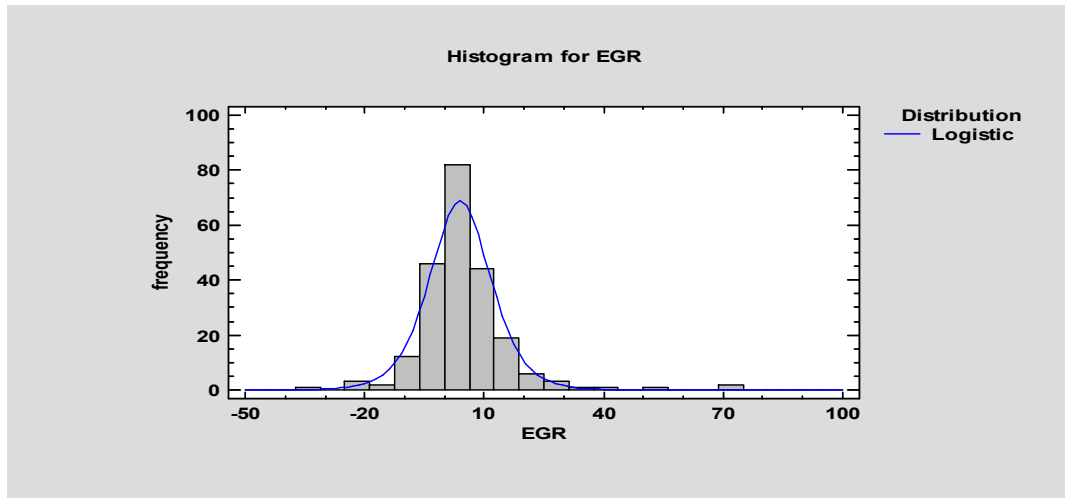

Fig. 4.3. Plot of logistic distribution on the histogram of quarterly economic growth rate
Source: Author's computation and Eviews 7.1 Output

Table 4.8. Goodness-of-Fit Test for EGR using Logistic Distribution

Logistic distribution				
Mean	**Standard deviation**	**Log likelihood**	**Kolmogorov-Smirnov D**	**P-Value**
4.78%	6.24%	-819.802	0.0949455	0.03588

Source: Author's computation and Eviews 7.1 Output

Fig. 4.3 shows the plot of logistic distribution on the histogram of quarterly economic growth of Nigeria. The graph shows that logistic distribution did not perfectly mirror the histogram of the variable. Also, from Table 4.8, the p-value of 0.3588 from the Kolmogorov-Smirnov test indicates that it is less than 0.05 significant levels. Therefore we conclude that logistic distribution is not a good fit for the quarterly economic growth rate data in Nigeria.

4.4.4 Cauchy distribution

Hypothesis IV

H_{04}**:** The quarterly economic growth rate of Nigeria follows the theoretical Cauchy distribution.

Fig. 4.4 shows the plot of Cauchy distribution on the histogram of quarterly economic growth of Nigeria. The graph depicts that Cauchy distribution did not mirror the histogram of the variable. Also, from Table 4.9, the p-value of 0.0395 from the Kolmogorov-Smirnov test shows that it is less than 0.05 significant levels. Therefore we conclude that Cauchy distribution is not a good fit for the quarterly economic growth rate data in Nigeria.
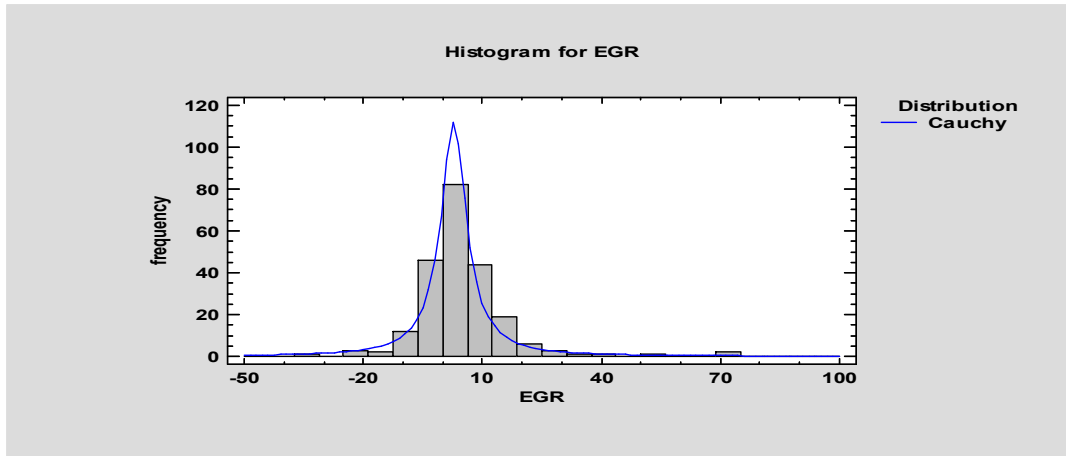
Fig. 4.4. Plot of Cauchy distribution on the histogram of quarterly economic growth rate
Source: Author's computation and Eviews 7.1 Output

Table 4.9. Goodness-of-fit test for EGR using Cauchy distribution

Cauchy distribution				
Mean	**Standard deviation**	**Log likelihood**	**Kolmogorov-Smirnov D**	**P-Value**
2.74%	3.95%	-810.948	0.0938037	0.0395

Source: Author's computation and Eviews 7.1 Output

4.4.5 Gumbel (largest extreme value) distribution

Hypothesis V

H_{05}**:** The quarterly economic growth rate of Nigeria follows the theoretical Gumbel (large extreme value) distribution
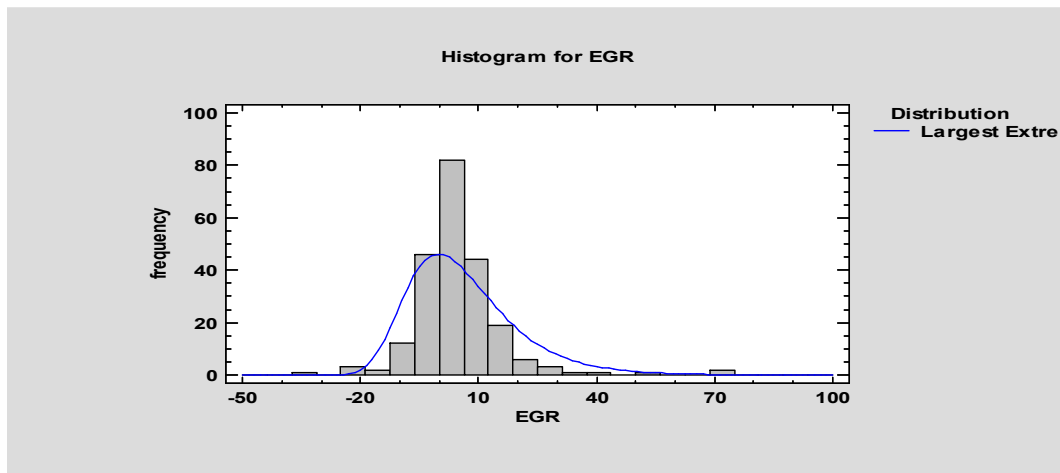

Fig. 4.5. Plot of largest extreme value distribution on the histogram of quarterly economic growth rate
Source: Author's computation and Eviews 7.1 Output

Table 4.10. Goodness-of-fit test for EGR using largest extreme value distribution

Largest extreme value distribution				
Mode	**Standard deviation**	**Log likelihood**	**Kolmogorov-Smirnov D**	**P-Value**
-0.16%	11.13%	-859.101	0.165237	0.0000

Source: Author's computation and Eviews 7.1 Output

Fig. 4.5 shows the plot of largest extreme value distribution on the histogram of quarterly economic growth of Nigeria. The graph depicts that largest extreme value distribution did not mirror the histogram of the variable. Also, from Table 4.10, the p-value of 0.0000 from the Kolmogorov-Smirnov test shows that it is less than 0.05 significant levels. Therefore we conclude that largest extreme value distribution is not a good fit for the quarterly economic growth rate data in Nigeria.

4.4.6 Generalized logistic distribution

Hypothesis VI

H_{06} : The quarterly economic growth rate of Nigeria follows the theoretical Generalized Logistic distribution
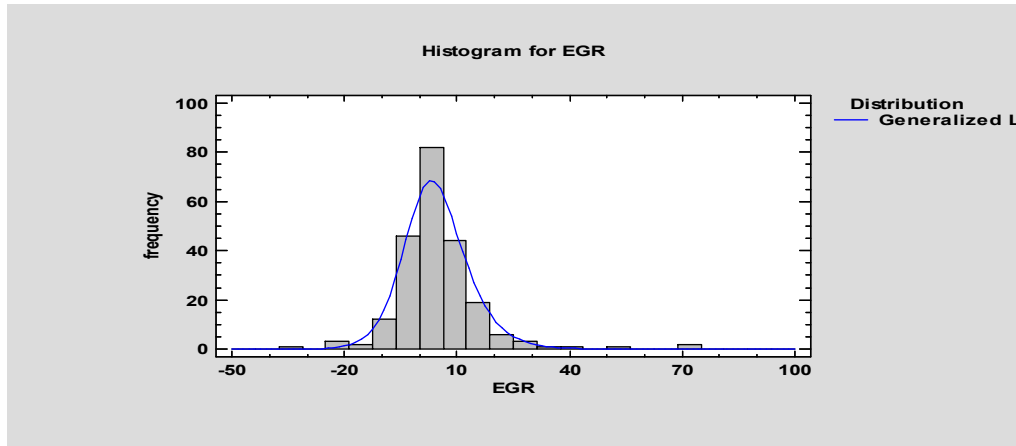

Fig. 4.6. Plot of generalized logistic distribution on the histogram of quarterly economic growth rate
Source: Author's computation and Eviews 7.1 Output

Table 4.11. Goodness-of-fit test for EGR using generalized logistic distribution

Generalized logistic distribution					
Location	**Scale**	**Shape**	**Log likelihood**	**Kolmogorov-Smirnov D**	**P-Value**
0.61%	5.72%	1.56%	-817.206	0.086536	0.0709

Source: Author's computation and Eviews 7.1 Output

Fig. 4.6 shows the plot of generalized logistic distribution on the histogram of quarterly economic growth of Nigeria. The graph portrays that generalized logistic distribution mirrored the histogram of the variable. Also, from Table 4.11, the p-value of 0.0709 from the Kolmogorov-Smirnov test shows that it is greater than 0.05 significant levels. Therefore we conclude that generalized logistic distribution is a good fit for the quarterly economic growth rate data in Nigeria.

4.5 Model selection

Two probability distributions, namely, Laplace distribution and generalized logistic distribution fitted the data of quarterly economic growth rate in Nigeria. The information criteria method was used to select the appropriate probability distribution that fits the data better between the two distributions.

From Table 4.12, the Laplace distribution is the most appropriate probability distribution that explains the behavior of quarterly economic growth rate of Nigeria since it has the least values of -9.33 and -2,51 for AIC and BIC respectively when compared to the generalized logistic distribution.

Therefore the appropriate probability density function of Laplace distribution becomes:

$$f(x) = \frac{1}{2(8.01)} \exp\left(-\left|\frac{x-4.78}{8.01}\right|\right) \quad -\infty < x < \infty$$

4.6 Probability estimation

The probability forecasts for Nigeria's economic growth rate in any quarter was computed using the probability density function of the best fit model, thus, the probability density function of Laplace distribution :

$$f(x) = \frac{1}{2(8.01)} \exp\left(-\left|\frac{x-4.78}{8.01}\right|\right) \quad -\infty < x < \infty$$

Table 4.12. The information criteria calculation for laplace distribution and generalized logistic distribution

Distribution	Number of parameters (k)	Number of observations (n)	Log likelihood (Lmax)	AIC	BIC
Laplace	2	223	-807.50	-9.33	-2.51
Generalized logistic	3	223	-817.206	-7.31	2.80

Source: Author's computation and Eviews 7.1 Output

Table 4.13. Probability forecasts for the quarterly economic growth rate (EGR) for Nigeria

Probability intervals	Description	Probabilities
$\Pr(x<0) = \int_{-\infty}^{0} f(x)dx$	Probability that Nigeria's economic growth rate in any quarter will be less than 0.	Pr = 0.28
$\Pr(0 \leq x \leq 9.9) = \int_{0}^{9.9} f(x)dx$	Probability that Nigeria's economic growth rate in any quarter will be positive single digit	Pr = 0.46
$\Pr(10 < x < 99.9) = \int_{10}^{99.9} f(x)dx$	Probability that Nigeria's economic growth rate in a quarter will be positive double digit:	Pr = 0.26

Source: Author's computation and Eviews 7.1 Output

Therefore, applying the probability density function, we obtained the probabilities in the Table 4.13.

The computed probabilities in Table 4.13 show that the chance of having a negative quarterly economic growth rate in Nigeria is 28%. While, to have a positive single digit quarterly economic growth rate the chance is 46%. In addition, to obtain a positive double digit quarterly economic growth rate is 26%.

5 Discussion of Results

The probability distribution modeling of Nigeria's economic growth was evaluated using the method of Goodness-of-fit. Six probability distributions were used for the modeling, namely, the Normal distribution, Laplace distribution, Logistic distribution, Cauchy distribution, Largest Extreme Value distribution and Generalized Logistic distribution by applying them on the quarterly economic growth rate data of Nigeria from 1960 to 2015.

5.1 Statistical tests

The data was subjected to several statistical tests to ascertain the data's behavior. From Table 4.2 it was shown that the data is stationary, this indicates that it is in equilibrium around its mean. Also, by applying the Jarque-Bera normality test, from Table 4.3 it showed that the data did not emerge from a normally distributed population which is typical of an economic data. In order to verify if the error variance satisfies the condition of homoscedasticity, the White's Heteroscedasticity test was conducted. From Table 4.4, the variance of the error was found to have a constant variance (homoscedastic). Consequently, the Breusch-Godfrey Serial Correlation Lm test was used to check for serial correlation among the values of quarterly economic growth rate data, from Table 4.5, it showed there was no serial correction existing among the values of the variable. This means that each value is independent one another.

5.2 Goodness-of-fit tests

The Smirnov-Kolmogorov goodness-of-test was used to assess the six probability distributions on how best they fit the quarterly economic growth rate data. It was found that two probability distributions fitted data. From Tables 4.7 and 4.11, these are distributions are, Laplace distribution and Generalized Logistic distribution with p-values of 0.0581 and 0.0709 respectively greater than the hypothesized significant level of 0.05. Hence, other probability distributions, specifically, Normal distribution, Logistic distribution, Cauchy distribution and Largest Extreme Value distributions did not fit the data.

5.3 Model selection

The best probability distribution model that fitted the quarterly economic growth rate data was selected using the method information criteria of Akaike Information Criteria (AIC) and Bayes Information Criteria (BIC). Accordingly, between the two probability distributions that fitted the data, Laplace distribution and Generalized Logistic distribution, the Laplace distribution was selected as the best fit probability distribution since both the AIC and BIC values were less than that of Generalized Logistic distribution. This is line with the works of Fagiolo, Napoletano and Roventini [1], Stockhammer and Oller [2], Podobnik, Horvatic, Njavro and Stanley [6] and Franke [8].

5.4 Probability forecasting

The probability estimations were computed using the selected best probability distribution model, Laplace distribution. Invariably, from the estimated probabilities, the chance of obtaining a negative quarterly economic growth is 28%. Therefore, since there is no serial correlation between the values of the variable, the chance of an economic recession, which is defined as a decline in economic growth rate for two or more consecutive quarters, is 8%. Also, the probability of having a positive single digit quarterly economic growth rate is 46%, this means that single digit economic growth rate is highly possible in Nigeria. In addition, having a double digit positive quarterly economic growth rate is 26%.

6 Conclusion

One of the functions of Central Bank of Nigeria is to maintain economic stability and growth in the country. In this regard, the monetary authority uses a time series tools to model Nigeria's economic growth rate for policy analysis and implementation. Therefore, the main interest of this study as stated in the statement of problem was to discover why despite several time series and econometric methods have been used to investigate the volatility of economic growth rate, yet none seems to explain the movement. Hence, this study has revealed that the Laplace distribution is the best probability distribution that explains behavior of the Nigeria's economic growth rate.

Consequently, this has exposed the reason for lack of consensus among researchers on the appropriate model to explain economic growth rate behavior in the country, which has resulted in the use of several time series and econometric methods to model economic growth rate of Nigeria. The findings from this work show that monetary authority in Nigeria can now use Laplace distribution as an alternative to model economic growth rate behavior.

Competing Interests

Authors have declared that no competing interests exist.

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