



# The Minimum Superior Dominating Energy of Graphs

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## Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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## Abstract

Kathiresan and Marimuthu were the pioneers of superior distance in graphs. The same authors put forth the concept of superior domination in 2008. Superior distance is the shortest walk between any two vertices including their respective neighbours. The minimum superior dominating energy  $\mathbb{E}_{sd}(G)$  is defined by the sum of the eigenvalues and it is obtained from the minimum superior dominating  $n \times n$  matrix  $\mathbb{A}_{sd}(G) = (e_{ij})$ . The minimum superior dominating energy for star and crown graphs are computed. Properties of eigenvalues of minimum superior dominating matrix for star, cocktail party, complete and crown graphs are discussed. Results related to upper and lower bounds of minimum superior dominating energy for star, cocktail party, complete and crown graphs are stated and proved.

**Keywords:** Minimum superior dominating set; minimum superior dominating energy; superior distance; superior neighbour; superior dominating eigenvalues.

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## 1 Introduction

Kathiresan and Marimuthu [1] were the pioneers of superior distance in graphs. Let  $D_{uv} = N[u] \cup N[v]$ . A  $D_{uv}$  walk is defined as a  $u - v$  walk in  $G$  that contains every vertex of  $D_{uv}$ . The superior distance  $d_s(u, v)$  from  $u$  to  $v$  is the length of the shortest  $D_{uv}$  walk. The superior neighbour of a vertex  $u$  is given by  $e_{sn}(u) = \min\{d_s(u, v) : v \in V(G) - \{u\}\}$ . A vertex  $v(\neq u)$  is called a superior neighbourhood vertex of  $u$  if  $d_s(u, v) = e_{sn}(u)$ . The same authors [2] “put forth the concept of superior domination in 2008. Superior distance is the shortest walk between any two vertices including their respective neighbours”. Different types of domination were also developed [3-4]. A subset  $D$  of  $V$  is said to be a dominating set, if every vertex not in  $D$  is adjacent to at least one vertex in  $D$ . Ivan Gutman [5] in 1978 introduced “energy of a graph”. Inspired by Gutman many authors have explored different types of energy in graph theory [6-10]. Kanna et al. [11] found “the minimum dominating energy of a graph”. Inspired by Kanna et al. [11] “minimum superior dominating energy  $\mathbb{E}_{sd}(G)$  of graphs is introduced in this paper, also  $\mathbb{E}_{sd}(G)$  of standard graphs, properties of  $\mathbb{E}_{sd}(G)$  and bounds of  $\mathbb{E}_{sd}(G)$  are studied”.

## 2 The Minimum Superior Dominating Energy

In this section, minimum superior dominating matrix  $\mathbb{A}_{sd}(G)$  and minimum superior dominating energy  $\mathbb{E}_{sd}(G)$  are defined. The  $\mathbb{E}_{sd}(G)$  of some standard graphs are obtained.

**Definition 2.1:** For  $G = (V, E)$  the superior neighbourhood vertex set of a vertex  $u$  is given by  $E_{sn}(u) = \{v / d_s(u, v) = e_{sn}(u) \forall u, v \in V(G)\}$ . Let  $D$  be a minimum superior dominating set of  $G$  then the minimum superior dominating matrix of  $G$  is a  $n \times n$  defined by  $\mathbb{A}_{sd}(G) = (e_{ij})$ , where

$$(e_{ij}) = \begin{cases} 1, & \text{if } v_i \in E_{sn}(v_j) \text{ or } v_j \in E_{sn}(v_i), \\ 1, & \text{if } i = j \text{ and } v_i \in D, \\ 0, & \text{otherwise} \end{cases}$$

**Definition 2.2:** The characteristic polynomial of  $\mathbb{A}_{sd}(G)$  is  $\mathcal{L}_n(G, \alpha) = \det(\mathbb{A}_{sd} - \alpha I)$ , where  $I$  is the identity matrix.

**Definition 2.3:** The minimum superior dominating eigenvalues of  $G$  are the eigenvalues of  $\mathbb{A}_{sd}(G)$ . Since  $\mathbb{A}_{sd}(G)$  is symmetric and real, the eigenvalues are real. The eigenvalues are arranged in non-increasing order  $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n$ .

**Definition 2.4:** The minimum superior dominating energy  $\mathbb{E}_{sd}(G)$  of  $G$  is defined by  $\mathbb{E}_{sd}(G) = \sum_{i=1}^n |\alpha_i|$ .

**Remark 2.1:** The trace of  $\mathbb{A}_{sd}(G)$  = Superior Domination number.

**Example 2.1:** The R graph consists of 6 vertices and 6 edges given in Fig. 1

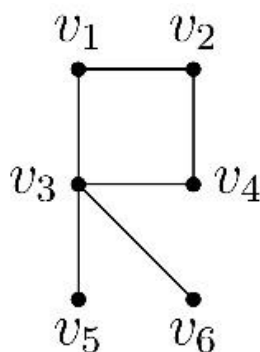


Fig. 1. R Graph

**Table 1. Superior neighbour distance and superior neighbour vertex set of R graph**

Vertex	Superior neighbour distance $d_s(v) = \min \{d_s(v, u) \forall u \in V(G)\}$	Superior neighbour vertex set $E_{sn}(v) = \{u/d_s(v, u) = e_{sn}(v) \forall u \in V(G)\}$
$v_1$	3	$\{v_2\}$
$v_2$	3	$\{v_1, v_4\}$
$v_3$	7	$\{v_1, v_4, v_5, v_6\}$
$v_4$	3	$\{v_2\}$
$v_5$	2	$\{v_6\}$
$v_6$	2	$\{v_5\}$

The minimum superior dominating sets of an R graph are  $D_1 = \{v_1, v_3\}$ ,  $D_2 = \{v_2, v_3\}$  and  $D_3 = \{v_3, v_4\}$ .

- $D_1 = \{v_1, v_3\}$ ,
- 

$$A_{sd}(G) = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial  $\mathcal{L}_n(G, \alpha) = \alpha^6 - 2\alpha^5 - 6\alpha^4 + 6\alpha^3 + 8\alpha^2 - 2\alpha - 1$ .

Minimum superior dominating eigenvalues are  $\alpha_1 \approx 3.0675, \alpha_2 \approx 1.5033, \alpha_3 \approx 0.4512, \alpha_4 \approx -0.2751, \alpha_5 \approx -1, \alpha_6 \approx -1.7469$ .

Minimum superior dominating energy  $\mathbb{E}_{sd}(G) \approx 8.044$ .

- $D_2 = \{v_2, v_3\}$ ,

$$A_{sd}(G) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial  $\mathcal{L}_n(G, \alpha) = \alpha^6 - 2\alpha^5 - 6\alpha^4 + 6\alpha^3 + 9\alpha^2$ .

Minimum superior dominating eigenvalues are  $\alpha_1 \approx 3, \alpha_2 \approx 1.7320, \alpha_3 \approx 0, \alpha_4 \approx 0, \alpha_5 \approx -1, \alpha_6 \approx -1.7320$ .

Minimum superior dominating energy  $\mathbb{E}_{sd}(G) \approx 7.464$ .

- $D_3 = \{v_3, v_4\}$ ,

$$A_{sd}(G) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial  $\mathcal{L}_n(G, \alpha) = \alpha^6 - 2\alpha^5 - 6\alpha^4 + 6\alpha^3 + 8\alpha^2 - 2\alpha - 1$ .

Minimum superior dominating eigenvalues are  $\alpha_1 \approx 3.0675, \alpha_2 \approx 1.5033, \alpha_3 \approx 0.4512, \alpha_4 \approx -0.2751, \alpha_5 \approx -1, \alpha_6 \approx -1.7469$ .

Minimum superior dominating energy  $\mathbb{E}_{sd}(G) \approx 8.044$ .

$\mathbb{E}_{sd}(G)$  of  $D_1$  and  $D_3$  is 8.044, but  $\mathbb{E}_{sd}(G)$  of  $D_2$  is 7.464. Therefore  $\mathbb{E}_{sd}(G)$  varies based on the superior dominating set.

**Remark 2.2:** The minimum superior dominating energy  $\mathbb{E}_{sd}(G)$  depends on the superior dominating set.

**Theorem 2.1:** For a star graph  $S_n$  where  $n \geq 2$  the minimum superior dominating energy of star  $\mathbb{E}_{sd}(S_n) = |-1|(n-2) + \left| \frac{(n-1)+\sqrt{(n-1)^2+4}}{2} \right| + \left| \frac{(n-1)-\sqrt{(n-1)^2+4}}{2} \right|$ .

Proof: Consider a star graph  $S_n$  with the vertex set  $V = \{v_1, v_2, \dots, v_n\}$ . The minimum superior dominating set is  $D = \{v_1\}$  where  $\deg(v_1) = \Delta(S_n)$  then

$$A_{sd}(S_n) = \begin{pmatrix} 1 & 1 & 1 & & 1 & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & 0 & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & 0 & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 & 1 \\ 1 & 1 & 1 & & 1 & 1 & 0 \end{pmatrix}_{n \times n}$$

Characteristic polynomial is  $\mathcal{L}_n(S_n, \alpha) = \det(A_{sd}(S_n) - \alpha I)$ .

$$= \begin{vmatrix} 1-\alpha & 1 & 1 & & 1 & 1 & 1 \\ 1 & -\alpha & 1 & \cdots & 1 & 1 & 1 \\ 1 & 1 & -\alpha & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & -\alpha & 1 & 1 \\ 1 & 1 & 1 & \cdots & 1 & -\alpha & 1 \\ 1 & 1 & 1 & & 1 & 1 & -\alpha \end{vmatrix}$$

The characteristic polynomial  $\mathcal{L}_n(S_n, \alpha) = (\alpha + 1)^{n-2}(\alpha^2 - (n-1)\alpha - 1)$

The minimum superior dominating eigenvalues are

$$\alpha = -1 \text{ ((n - 2) time),}$$

$$\alpha = \frac{(n-1)+\sqrt{(n-1)^2+4}}{2},$$

$$\alpha = \frac{(n-1)-\sqrt{(n-1)^2+4}}{2}.$$

The minimum superior dominating energy of the star  $S_n$  is given by

$$\mathbb{E}_{sd}(S_n) = |-1|(n-2) + \left| \frac{(n-1)+\sqrt{(n-1)^2+4}}{2} \right| + \left| \frac{(n-1)-\sqrt{(n-1)^2+4}}{2} \right|.$$

**Remark 2.3:** The energy of cocktail party and complete graph is same as star graph.

**Theorem 2.2:** For a crown graph  $H_n$  where  $n \geq 8$  the minimum superior dominating energy of crown

$$\mathbb{E}_{sd}(H_n) = \left| \frac{\left(\frac{n}{2}+1\right)+\sqrt{\left(\frac{n}{2}-1\right)^2+4}}{2} \right| + \left| \frac{\left(\frac{n}{2}+1\right)-\sqrt{\left(\frac{n}{2}-1\right)^2+4}}{2} \right| + \left| \frac{\left(1-\frac{n}{2}\right)+\sqrt{\left(\frac{n}{2}+1\right)^2-4}}{2} \right| + \left| \frac{\left(1-\frac{n}{2}\right)-\sqrt{\left(\frac{n}{2}+1\right)^2-4}}{2} \right|.$$

Proof: Let  $H_n$  be a crown graph with the vertex set  $V = \{v_1, v_2, \dots, v_n\}$ . The minimum superior dominating set is  $D = \{v_1, v_{\frac{n}{2}+1}\}$  then

$$\mathbb{A}_{sd}(H_n) = \begin{pmatrix} 1 & 0 & 0 & & 1 & 1 & 1 \\ 0 & 0 & 0 & \cdots & 1 & 1 & 1 \\ 0 & 0 & 0 & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & 1 & 0 & 0 \\ 1 & 1 & 1 & \cdots & 0 & 0 & 0 \\ 1 & 1 & 1 & & 0 & 0 & 0 \end{pmatrix}_{n \times n}$$

Characteristic polynomial is  $\mathcal{L}_n(H_n, \alpha) = \det (\mathbb{A}_{sd}(H_n) - \alpha I)$ .

$$= \begin{vmatrix} 1 - \alpha & 0 & 0 & & 1 & 1 & 1 \\ 0 & -\alpha & 0 & \cdots & 1 & 1 & 1 \\ 0 & 0 & -\alpha & & 1 & 1 & 1 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & 1 - \alpha & 0 & 0 \\ 1 & 1 & 1 & \cdots & 0 & -\alpha & 0 \\ 1 & 1 & 1 & & 0 & 0 & -\alpha \end{vmatrix}$$

The characteristic polynomial

$$\mathcal{L}_n(H_n, \alpha) = \alpha^{n-4} \left[ \alpha^2 - \left(\frac{n}{2} + 1\right) \alpha + \left(\frac{n}{2} - 1\right) \right] \left[ \alpha^2 + \left(\frac{n}{2} - 1\right) \alpha - \left(\frac{n}{2} - 1\right) \right]$$

The minimum superior dominating eigenvalues are

$$\alpha = 0 \text{ ((n - 4) time),}$$

$$\alpha = \frac{\left(\frac{n}{2} + 1\right) + \sqrt{\left(\frac{n}{2} - 1\right)^2 + 4}}{2},$$

$$\alpha = \frac{\left(\frac{n}{2} + 1\right) - \sqrt{\left(\frac{n}{2} - 1\right)^2 + 4}}{2},$$

$$\alpha = \frac{\left(1 - \frac{n}{2}\right) + \sqrt{\left(\frac{n}{2} + 1\right)^2 - 4}}{2},$$

$$\alpha = \frac{\left(1 - \frac{n}{2}\right) - \sqrt{\left(\frac{n}{2} + 1\right)^2 - 4}}{2}.$$

The minimum superior dominating energy of the crown  $H_n$  is given by

$$\mathbb{E}_{sd}(H_n) = \left| \frac{\left(\frac{n}{2} + 1\right) + \sqrt{\left(\frac{n}{2} - 1\right)^2 + 4}}{2} \right| + \left| \frac{\left(\frac{n}{2} + 1\right) - \sqrt{\left(\frac{n}{2} - 1\right)^2 + 4}}{2} \right| + \left| \frac{\left(1 - \frac{n}{2}\right) + \sqrt{\left(\frac{n}{2} + 1\right)^2 - 4}}{2} \right| + \left| \frac{\left(1 - \frac{n}{2}\right) - \sqrt{\left(\frac{n}{2} + 1\right)^2 - 4}}{2} \right|.$$

### 3 Properties of Minimum Superior Dominating Eigenvalues

In this section properties of eigenvalues of  $\mathbb{A}_{sd}(G)$  for star, cocktail party, complete and crown graphs are discussed. Bounds for minimum superior dominating energy  $\mathbb{E}_{sd}(G)$  of some standard graphs are obtained.

**Theorem 3.1:** If  $D$  is the minimum superior dominating set and  $\alpha_1, \alpha_2, \dots, \alpha_n$  are the eigenvalues of minimum superior dominating matrix  $\mathbb{A}_{sd}(G)$  then

1. For any graph  $G$ ,  $\sum_{i=1}^n \alpha_i = |D|$ ,
2. For a star graph  $S_n$ , cocktail party and complete graph,  $\sum_{i=1}^n \alpha_i^2 = |D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n - 1)$ .
3. For a crown graph  $H_n$ ,  $\sum_{i=1}^n \alpha_i^2 = |D| + \sum_{i=1}^n |E_{sn}(v_i)|$ .

Proof:

1. The trace of  $\mathbb{A}_{sd}(G)$  is the sum of eigenvalues of  $\mathbb{A}_{sd}(G)$ .

$$\sum_{i=1}^n \alpha_i = \sum_{i=1}^n e_{ii} = |D|.$$

2. For a star graph  $S_n$ , cocktail party and complete graph, the trace of  $[\mathbb{A}_{sd}(G)]^2$  is equal to the summation of the squares of eigenvalues of  $\mathbb{A}_{sd}(G)$ .

$$\sum_{i=1}^n \alpha_i^2 = \sum_{i=1}^n \sum_{j=1}^n e_{ij}e_{ij} = \sum_{i=1}^n (e_{ii})^2 + \sum_{i \neq j} e_{ij}e_{ij} = \sum_{i=1}^n (e_{ii})^2 + 2 \sum_{i < j} (e_{ij})^2$$

$$\sum_{i=1}^n \alpha_i^2 = |D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n - 1) \quad [Since, 2 \sum_{i < j} (e_{ij})^2 = \sum_{i=1}^n |E_{sn}(v_i)| + (n - 1)]$$

3. For a crown graph  $H_n$ , the trace of  $[\mathbb{A}_{sd}(H_n)]^2$  is equal to the summation of the squares of eigenvalues of  $\mathbb{A}_{sd}(H_n)$ .

$$\sum_{i=1}^n \alpha_i^2 = \sum_{i=1}^n \sum_{j=1}^n e_{ij}e_{ij} = \sum_{i=1}^n (e_{ii})^2 + \sum_{i \neq j} e_{ij}e_{ij} = \sum_{i=1}^n (e_{ii})^2 + 2 \sum_{i < j} (e_{ij})^2$$

$$\sum_{i=1}^n \alpha_i^2 = |D| + \sum_{i=1}^n |E_{sn}(v_i)| \quad [Since, 2 \sum_{i < j} (e_{ij})^2 = \sum_{i=1}^n |E_{sn}(v_i)|]$$

**Theorem 3.2:** For a star graph  $S_n$ , cocktail party and complete graph, if  $D$  be the minimum superior dominating set and  $W = |\det \mathbb{A}_{sd}(G)|$  then

$$\sqrt{|D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n - 1) + n(n - 1)W^{\frac{2}{n}}} \leq \mathbb{E}_{sd}(G) \leq \sqrt{n \left( \sum_{i=1}^n |E_{sn}(v_i)| + (n - 1) + |D| \right)}$$

Proof: By Cauchy schwarz inequality  $(\sum_{i=1}^n a_i b_i)^2 \leq (\sum_{i=1}^n a_i^2)(\sum_{i=1}^n b_i^2)$ . If  $a_i = 1$  and  $b_i = \alpha_i$  then

$$\left( \sum_{i=1}^n |\alpha_i| \right)^2 \leq \left( \sum_{i=1}^n 1 \right) \left( \sum_{i=1}^n \alpha_i^2 \right)$$

$$(\mathbb{E}_{sd}(G))^2 \leq n \left( |D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n - 1) \right)$$

$$\Rightarrow \mathbb{E}_{sd}(G) \leq \sqrt{n \left( |D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n - 1) \right)}$$

Since the arithmetic mean is not smaller than geometric mean

$$\frac{1}{n(n-1)} \sum_{i \neq j} |\alpha_i| |\alpha_j| \geq \left[ \prod_{i \neq j} |\alpha_i| |\alpha_j| \right]^{\frac{1}{n(n-1)}} = \left[ \prod_{i=1}^n |\alpha_i|^{2(n-1)} \right]^{\frac{1}{n(n-1)}} = \left[ \prod_{i=1}^n |\alpha_i| \right]^{\frac{2}{n}} = \left[ \prod_{i=1}^n \alpha_i \right]^{\frac{2}{n}}$$

$$\frac{1}{n(n-1)} \sum_{i \neq j} |\alpha_i| |\alpha_j| = |\det \mathbb{A}_{sd}(G)|^{\frac{2}{n}} = W^{\frac{2}{n}}$$

$$\sum_{i \neq j} |\alpha_i| |\alpha_j| \geq n(n-1)W^{\frac{2}{n}}$$

Now consider

$$(\mathbb{E}_{sd}(G))^2 = \left( \sum_{i=1}^n |\alpha_i| \right)^2 = \left( \sum_{i=1}^n |\alpha_i| \right)^2 + \sum_{i \neq j} |\alpha_i| |\alpha_j|$$

$$(\mathbb{E}_{sd}(G))^2 = \left( |D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n-1) \right) + n(n-1)W^{\frac{2}{n}}$$

$$\mathbb{E}_{sd}(G) \geq \sqrt{\left( |D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n-1) \right) + n(n-1)W^{\frac{2}{n}}}$$

**Theorem 3.3:** For a crown graph  $H_n$ , if  $D$  be the minimum superior dominating set and  $W = |\det \mathbb{A}_{sd}(H_n)|$  then

$$\sqrt{|D| + \sum_{i=1}^n |E_{sn}(v_i)| + n(n-1)W^{\frac{2}{n}}} \leq \mathbb{E}_{sd}(H_n) \leq \sqrt{n \left( \sum_{i=1}^n |E_{sn}(v_i)| + |D| \right)}$$

Proof: The proof is similar to Theorem 3.2.

**Theorem 3.4:** If  $\alpha_1(G)$  is the largest minimum superior dominating eigenvalue of  $\mathbb{A}_{sd}(G)$  then

1. For a star graph  $S_n$ , cocktail party and complete graph,  $\alpha_1(G) \geq \frac{|D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n-1)}{n}$
2. For a crown graph  $H_n$ ,  $\alpha_1(H_n) \geq \frac{|D| + \sum_{i=1}^n |E_{sn}(v_i)|}{n}$

Proof:

1. In a star graph  $S_n$ , cocktail party and complete graph, let  $Y$  be a non-zero vector, then by ref. [12],  $\alpha_1(\mathbb{A}_{sd}(G)) = \max_{Y \neq 0} \frac{Y' \mathbb{A}_{sd}(G) Y}{Y' Y}$

$$\alpha_1(\mathbb{A}_{sd}(G)) \geq \frac{U' \mathbb{A}_{sd}(G) U}{U' U} = \frac{|D| + \sum_{i=1}^n |E_{sn}(v_i)| + (n-1)}{n} \text{ where } U \text{ is the unit matrix.}$$

2. In a crown graph  $H_n$ , let  $Y$  be a non-zero vector, then by ref. [5],  $\alpha_1(\mathbb{A}_{sd}(H_n)) = \max_{Y \neq 0} \frac{Y' \mathbb{A}_{sd}(H_n) Y}{Y' Y}$

$$\alpha_1(\mathbb{A}_{sd}(H_n)) \geq \frac{U' \mathbb{A}_{sd}(H_n) U}{U' U} = \frac{|D| + \sum_{i=1}^n |E_{sn}(v_i)|}{n} \text{ where } U \text{ is the unit matrix.}$$

## 4 Conclusion

In this paper minimum superior dominating energy of graph is introduced. The minimum superior dominating energy for star and crown graphs are computed. The properties of eigenvalues of  $\mathbb{A}_{sd}(G)$  for star, cocktail party,

complete and crown graphs are discussed. Results related to the upper and lower bound of the energy of standard graphs is stated and proved.

## Competing Interests

Authors have declared that no competing interests exist.

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