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On Application of Two- Stage Stochastic Fully Fuzzy Linear Programming for Water Resources Management Optimization

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this paper, a two- stage stochastic fully fuzzy linear programming is developed for a management problem in terms of water resources allocations to illustrate the applicability of a proposed approach. A proposed approach converts the problem into a triple- objective problem and then a weighting method is utilized for solving it. The advantage of the approach is to generate a set of solutions for water resources planning which help the decision maker to make tradeoffs between the efficiency of economic and the risk violation of the constrains. A case study is given for illustration.

Keywords: Water resources; policy analysis; uncertainty; two- stage; multi- objective programming; weighting method.

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1 Introduction

Water Management is defined as the control and movement of Meta resources that is to minimize the damage to life and property and to maximize efficient beneficial use [1]. One advantage of the good water management of dams and levees is to reduce of the risk of harm which due to flooding [1]. It is clear that the irrigate water management systems make the limited water supplies for agriculture most efficient used.

A long with development and regulation, Kirpich [2] investigated another phrase (Holistic water management) which used to outline water management. Grigg [3] introduced a brief of integrated water resources management (IWRM) definition. As known, fuzzy set theory was introduced by Zadeh [4] to deal with fuzziness. Up to now, fuzzy set theory has been applied to broad fields. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers). For the fuzzy set theory development, we may referee to the papers of Kaufmann [5], and Dubois and Prade [6], they extended the use of algebraic operations of real numbers to fuzzy numbers by the use a fuzzifaction principle. Dubois and Prade [6] studied fuzzy linear constraints with fuzzy numbers. Lu et al. [7] introduced the definition of an inexact rough interval fuzzy linear programming method and investigated for generating water allocation to the agricultural irrigation system. Shaocheng [8] studied two kinds of linear programming problems with fuzzy numbers called: interval numbers and fuzzy number linear programming, respectively. Tanaka et al. [9] have formulated and proposed a method for solving fuzzy coefficients linear programming. Wang and Qiao [10] put forward a model of linear programming with fuzzy random variable coefficients. Through a two- stage dynamic programming approach, Ferrero et al. [11] have examined a long- term hydrothermal scheduling of multi- reservoir systems. Bellman and Zadeh [12] introduced the concept of a maximizing decision-making problem. Zhao et al. [13] introduced the complete solution set for the fuzzy linear programming problems using linear and nonlinear membership functions. For water resources management. An enormous of optimization techniques have been developed (Slowinski [14]; Wu et al. [15]; Jairaj and Vedula [16]; Maqsood et al. [17], Xu et al. [18], Wang and Adams [19], and Wang et al. [20]. To quantify the economic trade- offs when reducing groundwater abstraction to sustainable level, Mortinsen et al. [21] applied a multi- objective multi- temporal deterministic hydro economic optimization approach for this purpose. Veeramani et al. [22] studied fuzzy MOLP problem with fuzzy technological coefficients and resources. Kiruthiga and Loganathan [23] reduced the Fuzzy MOLP problem to the corresponding ordinary one using the ranking function and hence solved it using the fuzzy programming technique. Hamadameen [24] proposed a technique for solving fuzzy MOLP problem in which the objective functions coefficients are triangular fuzzy numbers. Khalifa [25] studied a water resources management problem as an application of a two- stage fuzzy random programming.

The remainder of the paper is as: Some preliminaries are introduced in section2. In section 3, a management problem in terms of water resources allocations is presented. A solution procedure for solving the problem is considered in section4. In section 5, a numerical example is given for illustration. Finally, some concluding remarks are reported in section 6.

2 Preliminaries

Some of basic concepts and related results to fuzzy numbers and some of their arithmetic operations, triangular fuzzy numbers and some of algebraic operations are reviewed in this section.

(Kaufmann and Gupta [26], Sakawa [27], Zimmermann [28], and Liang et al. [29]).

Definition1. A fuzzy number $\widetilde{p}$ is a mapping

$\mu_{\widetilde{p}} : R \to [0, 1]$, with the following properties:

(i) $\mu_{\widetilde{p}}(x)$ is an upper semi- continuous membership function;

(ii) $\widetilde{p}$ is a convex set, i. e., $\mu_{\widetilde{p}}(\lambda x+(1-\lambda)y) \geq \min\{\mu_{\widetilde{p}}(x), \mu_{\widetilde{p}}(y)\}$ for all $x, y \in R, 0 \leq \lambda \leq 1$;

(iii) $\widetilde{p}$ is normal, i. e., $\exists x_0 \in R$ for which $\mu_{\widetilde{p}}(x) = 1$;

(iv) Supp $(\widetilde{p}) = \{x : \mu_{\widetilde{p}}(x) > 0\}$ is the support of a fuzzy set $\widetilde{p}$.

Definition2. A triangular fuzzy number (T.F.N.) is denoted by $\widetilde{A} = (a_1, a_2, a_3)$, and having membership function defined as:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 0, & x > a_3. \end{cases}$$

Also, T.F.N. parametric form for level α can be characterized as:

$$\begin{aligned} \widetilde{A}_\alpha &= [(a_2-a_1)\alpha + a_1, -(a_3-a_2)\alpha + a_3] \\ &= [\underline{v}(\alpha), \bar{v}(\alpha)]; \forall 0 < \alpha \leq 1. \end{aligned}$$

Definition3. A T.F.N. $\widetilde{A} = (a_1, a_2, a_3)$ is called non- negative triangular fuzzy number if $a_1 \geq 0$.

Definition4. Let $\widetilde{A} = (a_1, a_2, a_3) \geq \widetilde{0}$, and $\widetilde{B} = (b_1, b_2, b_3) \geq \widetilde{0}$, the formulas for the addition, subtraction, scalar multiplication, and multiplication can be defined:

1. Addition:

$$\begin{aligned} &\widetilde{A} \oplus \widetilde{B} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) \\ &= (a_1+b_1, a_2+b, a_3+b_3). \end{aligned}$$

2. Subtraction:

$$\begin{aligned} &\widetilde{A} (-) \widetilde{B} = (a_1, a_2, a_3)(-) (b_1, b_2, b_3) \\ &= (a_1-b_3, a_2-b, a_3-b_1). \end{aligned}$$

3. Multiplication:

$$\widetilde{A} \otimes \widetilde{B} = \begin{cases} (a_1b_1, a_2b_2, a_3b_3), & a_1 \geq 0 \\ (a_1b_3, a_2b_2, a_3b_1), & a_1 < 0, a_3 \geq 0 \\ (a_1b_3, a_2b_2, a_3b_1), & a_3 < 0 \end{cases}$$

Remark1. $\widetilde{A} \geq \widetilde{0}$ if and only if $a_1 \geq 0, a_1 - a_2 \geq 0, a_1 + a_3 \geq 0.$

Definition5. A trapezoidal fuzzy number (Tr.F.N.) is denoted by $\widetilde{A} = (a_1, a_2, a_3, a_4)$, and have membership defined as:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 0, & x > a_4 \end{cases}$$

Also, a Tr. F.N., can be characterized by its interval of confidence (or parametric form) at interval of confidence at level α is defined by Liang et al. [29]:

$$\begin{aligned} \widetilde{A}_\alpha &= [(a_2 - a_1)\alpha + a_1, -(a_4 - a_3)\alpha + a_4] \\ &= [\underline{v}(\alpha), \bar{v}(\alpha)]; \ \forall\, 0 < \alpha \leq 1. \end{aligned}$$

Remark2. A Tr. F.N $\widetilde{A} = (a_1, a_2, a_3, a_4)$ can also signify a T.F.N. $\widetilde{v} = (a_1, a_2, a_3) = ((v)^-, (v)^c, (v)^+)$, if $a_2 = a_3$.

3 Problem Formulation and Solution Concepts

A typical water resources management problem [30,31] as follows

$$\max f = \sum_{j=1}^{n} \widetilde{B}_j T_j - E\left[\sum_{j=1}^{n} C_j\, S_{jQ}\right] \tag{1}$$

Subject to

$$\sum_{j=1}^{n} \left(T_j - S_{jQ}\right)\left(1 + \delta\right) \leq Q \quad \text{(Water availability constraints)} \tag{2}$$

$$S_{jQ} \leq T_j \leq T_{j\max}; \forall\, j, \quad \text{(Water- allocation target constraints)} \tag{3}$$

$$S_{jQ} \geq 0; \forall\, j \quad \text{(Non- negative and technical constraints)} \tag{4}$$

Where,

f : A benefit of system ($);

B_j : Net benefit to user j per m^3 of water allocated ($/m^3) / (First- stage revenue parameters)

T_j: Allocation target for water that is promised to user j (m^3) /(First- Stage decision variables)
$E[.]$: Expected value of a random variable;
C_j : Loss to user j per m^3 of water not delivered, $C_j > NB_j$ (\$/m^3)/ (Second- Stage cost parameters)
S_{jQ} : Shortage of water to user j when the seasonal flow is Q (m^3)
(Second- Stage decision variables))
Q : Total amount of seasonal flow (m^3) (random variables);
δ : Rate of water loss during transportation;
$T_{j\max}$: Maximum allowable allocation amount for user j (m^3);
m : Total number of water users;
i : Water user, $i = 1, 2, 3$, where $i = 1$ for municipality, $i = 2$ for the industrial user, and $i = 3$
for the agricultural sector

Referring to Huang and Loucks [30], the above problem can be reformulated as in the following form

$$\max f = \sum_{j=1}^{n} B_j T_j - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i C_j S_{ij} \tag{5}$$

Subject to

$$\sum_{j=1}^{n} \left(T_j - S_{ij}\right)\left(1 + \delta\right) \le q_i ; \forall i, \quad \text{(Water availability constraints)} \tag{6}$$

$$S_{ij} \le T_j \le T_{j\max} ; \forall, j \text{ (Water- allocation target constraints)} \tag{7}$$

$$S_{ij} \ge 0; \forall i, j \quad \text{(Non- negative and technical constraints)} \tag{8}$$

Where, S_{ij} is the amount by which the water- allocation target T_j is not met when the seasonal flow is q_i with probability p_i.

Consider the fuzzy model for the problem (5)- (8) as

$$\max \widetilde{f} = \sum_{j=1}^{n} \widetilde{B}_j \otimes \widetilde{T}_j - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i \left(\widetilde{C}_j \otimes \widetilde{S}_{ij}\right) \tag{9}$$

Subject to

$$\sum_{j=1}^{n} \left(\widetilde{T}_j - \widetilde{S}_{ij}\right)\left(1 + \widetilde{\delta}\right) \le \widetilde{q}_i ; \forall i, \tag{10}$$

$$\widetilde{S}_{ij} (\le) \widetilde{T}_j (\le) \widetilde{T}_{j\max} ; \forall, j \tag{11}$$

$$\widetilde{S}_{ij} \ge \widetilde{0}; \forall i, j \tag{12}$$

Where, $\widetilde{B}_j$, $\widetilde{C}_j$, $\widetilde{\delta}$, $\widetilde{q}_i$, $T_{j\max}$; $\widetilde{T}_j$, and $\widetilde{S}_{ij}$; $\forall\, i, j$ are triangular fuzzy parameters and variables.

Definition 6. (Optimal fuzzy solution). The $\left(\widetilde{S}_{ij}\right)^*$ which satisfy the conditions in (10)- (12) is called a fuzzy optimization solution .

4 Solution Procedure

The steps of the solution procedure for solving the problem (9)- (12) are given as in the following steps:

Step 1: Convert problem (9)- (12) into the corresponding problem using the arithmetic operations of T.F.N.

$$\max \widetilde{f} = \sum_{j=1}^{n}\left((B_j\,T_j)^l,(B_j\,T_j)^c,(B_j\,T_j)^u\right) - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left((C_j\,S_{ij})^l,(C_j\,S_{ij})^c,(C_j\,S_{ij})^u\right) \quad (13)$$

Subject to

$$\left(\begin{array}{l} \sum_{j=1}^{n}\left((T_j)^l-(S_{ij})^l,\,(T_j)^c-(S_{ij})^c,(T_j)^u-(S_{ij})^u\right)\left(1+\delta^l,1+\delta^c,1+\delta^u\right)\le \\ \qquad\left((q_i)^l,(q_i)^c,(q_i)^u\right);\forall i, \end{array}\right) \quad (14)$$

$$\left((S_{ij})^l,(S_{ij})^c,(S_{ij})^u\right)(\le)\left((T_j)^l,(T_j)^c,(T_j)^u\right)(\le)\left((T_{j\max})^l,(T_{j\max})^c,(T_{j\max})^u\right);\forall, j \quad (15)$$

$$\left((S_{ij})^l,(S_{ij})^c,(S_{ij})^u\right)\ge \widetilde{0};\forall\, i, j \quad (16)$$

Problem (13)- (16) can be rewritten as

$$\max \widetilde{f} = \sum_{j=1}^{n}\left((B_j\,T_j)^l,(B_j\,T_j)^c,(B_j\,T_j)^u\right) - \sum_{i=1}^{m}\sum_{j=1}^{n} p_i\left((C_j\,S_{ij})^l,(C_j\,S_{ij})^c,(C_j\,S_{ij})^u\right) \quad (17)$$

$$M = \left\{\begin{array}{l} \sum_{j=1}^{n}\left((T_j)^l-(S_{ij})^l\right)\left(1+\delta^l\right)\le (q_i)^l;\forall i \\ \sum_{j=1}^{n}\left((T_j)^c-(S_{ij})^c\right)\left(1+\delta^c\right)\le (q_i)^c;\forall i \\ \sum_{j=1}^{n}\left((T_j)^u-(S_{ij})^u\right)\left(1+\delta^u\right)\le (q_i)^u;\forall i \\ (S_{ij})^l(\le)(T_j)^l(\le)(T_{j\max})^l;\forall, j \\ (S_{ij})^c(\le)(T_j)^c(\le)(T_{j\max})^c;\forall, j \\ (S_{ij})^u(\le)(T_j)^u(\le)(T_{j\max})^u;\forall, j \\ (S_{ij})^l\ge 0,\,(S_{ij})^c-(S_{ij})^l\ge 0,(S_{ij})^u-(S_{ij})^c\ge 0;\forall\, i, j \end{array}\right. \quad (18)$$

Step2: Referring to the problem (17) - (18), it can be converted into

$$
\begin{aligned}
&\max f_1 = \left(\sum_{j=1}^{n} \left(B_j\, T_j\right)^c - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i \left(C_j\, S_{ij}\right)^c \right) \\
&\min f_2 = \left(\sum_{j=1}^{n} \left(\left(B_j\, T_j\right)^u - \left(B_j\, T_j\right)^l \right) - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i \left(\left(C_j\, S_{ij}\right)^u - \left(C_j\, S_{ij}\right)^l \right) \right) \\
&\max f_3 = \left(\sum_{j=1}^{n} \left(\left(B_j\, T_j\right)^u + \left(B_j\, T_j\right)^l \right) - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i \left(\left(C_j\, S_{ij}\right)^u + \left(C_j\, S_{ij}\right)^l \right) \right) \\
&\text{Subject to: } \left(\left(S_{ij}\right)^c, \left(S_{ij}\right)^l, \left(S_{ij}\right)^u \right) \in M
\end{aligned}
\tag{19}
$$

.

Step3: Solve problem (19) by using the weighting method (Zadeh [32])

$$
\begin{aligned}
&\max \left(0.5 f_1 - 0.2 f_2 + 0.3 f_3\right) \\
&\text{Subject to} \\
&\left(\left(S_{ij}\right)^c, \left(S_{ij}\right)^l, \left(S_{ij}\right)^u \right) \in M; \forall i, j
\end{aligned}
\tag{20}
$$

5 Numerical Example

Consider the problem introduced by Wang and Huang [31] involving triangular fuzzy numbers as

Table 1. Economic data (\$/ m^3) and seasonal flows (in 10^6 m^3) under different probability levels

Activity	User		
	Municipal **$(i=1)$**	**Industrial** **$(i=2)$**	**Agricultural** $(i=3)$
Maximum allowable allocation $\left(\tilde{T}_{\max}\right)$	$(7,8,9)$	$(7,8,10)$	$(7,8,9)$
Target of water allocation $\left(\tilde{T}_j\right)$	$(1,2,3)$	$(2,3,5)$	$(2,4,5)$
Net benefit when water demand is satisfied $\left(\tilde{B}_j\right)$	$(95,100,110)$	$(45,50,70)$	$(28,30,33)$
Reduction of the net benefit when $\left(\tilde{C}_j\right)$ Continue	$(220,250,285)$	$(55,60,90)$	$(45,50,75)$
Flow level	**Probability**		**Seasonal flow(%)**
Low ($i=1$)	0.3		$(2,3,4)$
Medium ($i=2$)	0.5		$(7,9,13)$
High ($i=3$)	0.2		$(14,16,20)$
Water loss ($\tilde{\delta}$)			$(0.15,0.20,0.40)$

Putting the above values in the MOLP problem (20) as

$$
\max \left(
\begin{aligned}
&484 - 3.75\left(S_{11}\right)^c - 62.5\left(S_{21}\right)^c - 25\left(S_{31}\right)^c - 9\left(S_{12}\right)^c - 15\left(S_{22}\right)^c - 6\left(S_{32}\right)^c \\
&- 7.5\left(S_{13}\right)^c - 12.5\left(S_{23}\right)^c - 5\left(S_{33}\right)^c - 8.55\left(S_{11}\right)^u - 33\left(S_{11}\right)^l - 14.25\left(S_{21}\right)^u \\
&- 55\left(S_{21}\right)^l - 5.7\left(S_{31}\right)^u - 22\left(S_{31}\right)^l - 2.7\left(S_{12}\right)^u - 8.25\left(S_{12}\right)^l - 4.5\left(S_{22}\right)^u \\
&- 13.75\left(S_{22}\right)^l - 1.8\left(S_{32}\right)^u - 5.5\left(S_{32}\right)^l - .2.25\left(S_{13}\right)^u - 6.75\left(S_{13}\right)^l \\
&- 3.75\left(S_{23}\right)^u - 11.25\left(S_{23}\right)^l - 1.5\left(S_{33}\right)^u - 4.4\left(S_{33}\right)^l
\end{aligned}
\right)
$$

Subject to

$$(S_{11})^l + (S_{12})^l + (S_{13})^l \geq 4.9826,$$
$$(S_{21})^l + (S_{22})^l + (S_{23})^l \geq 4.9391,$$
$$(S_{31})^l + (S_{32})^l + (S_{33})^l \geq 4.8783,$$
$$(S_{11})^c + (S_{12})^c + (S_{13})^c \geq 8.975,$$
$$(S_{21})^c + (S_{22})^c + (S_{23})^c \geq 8.925,$$
$$(S_{31})^c + (S_{32})^c + (S_{33})^c \geq 8.8667,$$
$$(S_{11})^u + (S_{12})^u + (S_{13})^u \geq 12.97,$$
$$(S_{21})^u + (S_{22})^u + (S_{23})^u \geq 12.907,$$
$$(S_{31})^u + (S_{32})^u + (S_{33})^u \geq 12.857,$$
$$0 \leq (S_{11})^u - (S_{11})^c \leq 1;$$
$$0 \leq (S_{12})^u - (S_{12})^c \leq 1;$$
$$0 \leq (S_{13})^u - (S_{13})^c \leq 1,$$
$$0 \leq (S_{21})^u - (S_{21})^c \leq 1;$$
$$0 \leq (S_{22})^u - (S_{22})^c \leq 1;$$
$$0 \leq (S_{23})^u - (S_{23})^c \leq 1,$$
$$0 \leq (S_{31})^u - (S_{31})^c \leq 1;$$
$$0 \leq (S_{32})^u - (S_{32})^c \leq 1;$$
$$0 \leq (S_{33})^u - (S_{33})^c \leq 1;$$
$$0 \leq (S_{11})^l, (S_{21})^l, (S_{31})^l;$$
$$(S_{12})^l, (S_{22})^l, (S_{32})^l;$$
$$(S_{13})^l, (S_{23})^l, (S_{33})^l \leq 1.$$

The solution of the problem obtained by using the Lingo computer package as in the following table

Table 2. Optimal fuzzy solution

Variable	Optimal fuzzy value	
$\tilde{S}_{11}$	$(0,1,2)$	Optimum fuzzy value $\tilde{f}^* = (0,0,502.2849)$
$\tilde{S}_{12}$	$(0,1,2)$	
$\tilde{S}_{13}$	$(6.97,7.97,9.97)$	
$\tilde{S}_{21}$	$(0,0,1)$	
$\tilde{S}_{22}$	$(0,1,2)$	
$\tilde{S}_{23}$	$(7.907,8.907,9.097)$	
$\tilde{S}_{31}$	$(0,0,1)$	
$\tilde{S}_{32}$	$(0,1,2)$	
$\tilde{S}_{33}$	$(7.857,8.857,9.957)$	

6 Conclusions

The studying of fully fuzzy linear programming for water resources management problem due to its close connection with human life, which is considered great importance. The advantages of two- stage stochastic fully fuzzy linear programming framwark are:

- Handle dual uncertainties as triangular fuzzy numbers,
- Provides the decision makers (DMs) with the information about the risk, and
- Enables the DM to quantify the relationship between the value of the objective function and the violating risk.

A proposed approach has been used to convert the problem under study into a multi- objective problem. The advantage of the approach is significant for being used in interactive methods for making any comment by related managers and achieving the solutions logically.

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Competing Interests

Authors have declared that no competing interests exist.

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